HW #10 Solutions

4.2.2) (a) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

(b) $4^{1.5} = 4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

(c) $9^{-0.5} = \frac{1}{9^{0.5}} = \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

4.2.10) (a) $\ln\left(\frac{\sqrt[3]{c}}{ab}\right) = \ln \sqrt[3]{c} + (-\ln a) - \ln b = \frac{1}{3}t - r - s$

(b) $\ln\sqrt{\frac{ab^3}{c^2}} = \ln \frac{(ab^3)^{\frac{1}{2}}}{c} = \ln a^{\frac{1}{2}} + \ln b^{3/2} - \ln c = \frac{1}{2}r + \frac{3}{2}s - t$

4.2.14) $\frac{1}{2}\log x - 3\log(\sin 2x) + 2 = \log x^{1/2} - \log(\sin 2x)^3 + \log 100$

$= \log \frac{100\sqrt{x}}{(\sin 2x)^3}$

4.2.26) $3^x = 2 \qquad \ln 3^x = \ln 2 \qquad x \ln 3 = \ln 2 \qquad x = \frac{\ln 2}{\ln 3}$

4.2.32) $e^{2x} - e^x = 6 \longrightarrow u^2 - u = 6 \longrightarrow u^2 - u - 6 = 0$

$(u-3)(u+2) = 0 \longrightarrow u = 3, -2 \longrightarrow e^x = 3,\ e^x = -2$

$x = \ln 3$ $\qquad$ ~~$x = \ln(-2)$~~ Impossible

4.2.42) $D = 0.051517\,(1.1306727)^x \longrightarrow \log D = \log(0.051517) + x\log(1.1306727)$

$\log D - \log(0.051517) = x \log(1.1306727)$

$x = \frac{\log D - \log(0.051517)}{\log(1.1306727)}$ $\qquad$ In this case, we want $D = 1000$, so $\log D = 3$

$x = 80.4$

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4.2.18) $\ln(x^2) = 4$ $\quad e^{\ln(x^2)} = e^4$

$x^2 = e^4$ $\quad x = e^2$

4.2.28) $3e^{-2x} = 5$ $\quad e^{-2x} = 5/3$ $\quad -2x = \ln(5/3)$

$x = \frac{-\ln(5/3)}{2}$

4.2.48) $Q(t) = 12e^{-0.055t}$

(a) $Q(0) = 12e^{-(0.055)(0)} = 12e^0 = 12$ grams

(b) $Q(4) = 12e^{-(0.055)(4)} = 9.63$ g

(c) $6 = 12e^{-(0.055t)}$

$\frac{1}{2} = e^{-0.055t} \longrightarrow \ln\frac{1}{2} = -0.055t \rightarrow t = \frac{-\ln(1/2)}{0.055} \approx 12.6$ hours

4.2.49) (a) $pH = -\log(3.9 \times 10^{-8}) = 7.4$ basic

(b) $pH = -\log(6.3 \times 10^{-5}) = 4.2$ acidic

(c) $pH = -\log(4.0 \times 10^{-7}) = 6.4$ acidic

(d) $pH = -\log(1.2 \times 10^{-1}) = 5.9$ acidic

4.2.55) (a) $\log E = 4.4 + 1.5(8.2) = 16.7$

$E = 10^{16.7} = 5.01 \times 10^{16}$ J

(b) $\log E = 4.4 + 1.5M \longrightarrow M_1 = \frac{\log E - 4.4}{1.5}$

$M_2 = \frac{\log(10E) - 4.4}{1.5} = \frac{\log 10 + \log E - 4.4}{1.5} = \frac{1 + \log E - 4.4}{1.5}$

$M_2 - M_1 = \frac{1 + \log E - 4.4}{1.5} - \frac{\log E - 4.4}{1.5} = \frac{1}{1.5} = \boxed{\frac{2}{3}}$