Solution Set 32

Section 9.8

(3) $\int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = \lim_{b\to 0}\left(-e^{-b} - (-1)\right) = 0 - -1 = 1$

(4) $u = 1+x^2$
$du = 2x\,dx$

$\frac{1}{2}\int_{-1}^{\infty} \frac{2x\,dx}{1+x^2} = \frac{1}{2}\int_{2}^{\infty} \frac{1}{u}\,du = \frac{1}{2}\ln u\Big|_2^\infty$

$= \lim_{b\to\infty} \frac{1}{2}\left(\ln b - \ln 2\right) = \infty$ diverges

(6) $\int_0^\infty x e^{-x^2}\,dx$ $u = -x^2$
$du = -2x\,dx$

$-\frac{1}{2}\int_0^\infty (e^{-x^2})(-2x\,dx) = -\frac{1}{2}\int_0^{-\infty} e^u\,du$

$= \lim_{b\to\infty} -\frac{1}{2}\left(e^{b} - e^{0}\right) = -\frac{1}{2}(0-1) = \frac{1}{2}$

(8) $\int_2^\infty \frac{1}{x\sqrt{\ln x}}\,dx$ $u = \ln x$
$du = \frac{1}{x}\,dx$

$\int_{\ln 2}^{\infty} u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}\Big|_{\ln 2}^{\infty}$

$= \lim_{b\to\infty} \left(2(b)^{\frac{1}{2}} - 2(\ln 2)^{\frac{1}{2}}\right)$ diverges

(12) $u = 3-2e^x$
$du = -2e^x\,dx$

$\int \frac{e^x\,dx}{3-2e^x} = -\frac{1}{2}\int \frac{-2e^x\,dx}{3-2e^x} = -\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}\ln u + C$

$\Rightarrow \int_{-\infty}^{0} \frac{e^x\,dx}{3-2e^x} = -\frac{1}{2}\ln(3-2e^x)\Big|_{-\infty}^{0}$

$= \lim_{a\to-\infty} -\frac{1}{2}\ln(3-2e^0) + \frac{1}{2}\ln(3-2e^a) = -\frac{1}{2}\ln(1) + \frac{1}{2}\ln(3)$

$= \frac{1}{2}\ln(3)$

(14) $u = x^2+2$
$du = 2x\,dx$

$$\int \frac{x}{\sqrt{x^2+2}}\,dx = \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2+2}} = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}(2u^{\frac{1}{2}}) + c$$

$$= u^{\frac{1}{2}} + c = \sqrt{x^2+2} + c$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx = \sqrt{x^2+2}\,\Big|_{-\infty}^{\infty} = \lim_{a\to\infty} \sqrt{x^2+2}\,\Big|_{-a}^{a}$$

$$= \lim_{a\to\infty} \sqrt{a^2+2} - \sqrt{(-a)^2+2} = \lim_{a\to\infty} \sqrt{a^2+2} - \sqrt{a^2+2} = 0$$

You can also evaluate the integral $\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx$ by noticing that the integrand is an odd function and the limits of integration are symmetric about the origin. This means that the integral must be equal to zero.

(22) $u = 9-x^2$
$du = -2x\,dx$

$$\int_{-3}^{1} \frac{x\,dx}{\sqrt{9-x^2}} = -\frac{1}{2}\int_{-3}^{1} \frac{-2x\,dx}{\sqrt{9-x^2}} = -\frac{1}{2}\int_{0}^{8} u^{-\frac{1}{2}}\,du$$

$$= -u^{\frac{1}{2}}\Big|_{0}^{8} = -\sqrt{8} + \sqrt{0} = -\sqrt{8}$$