HW # 30

7.9

3. a. $\int_1^{ac} \frac{1}{t}\, dt = \ln t \Big|_1^{ac} = \ln ac = \ln a + \ln c = \boxed{7}$

b. $\int_1^{1/c} \frac{1}{t}\, dt = \ln {}^1\!/_c = \ln 1 - \ln c = \boxed{-5}$

c. $\int_1^{a/c} \frac{1}{t}\, dt = \ln \frac{a}{c} = \ln a - \ln c = \boxed{-3}$

d. $\int_1^{a^3} \frac{1}{t}\, dt = \ln a^3 = 3 \ln a = \boxed{6}$

12. a. $\lim_{x \to +\infty} \left(1 + \frac{1}{3x}\right)^x$

$u = 3x,\ x = \frac{1}{3}u \longrightarrow \lim_{u \to +\infty} \left(1 + \frac{1}{u}\right)^{\frac{1}{3}u} = \lim \left[\left(1+\frac{1}{u}\right)^u\right]^{1/3}$

$= \boxed{e^{1/3}}$

b. $\lim_{x \to 0} (1+x)^{\frac{1}{3x}} = \lim_{x \to 0} \left[(1+x)^{\frac{1}{x}}\right]^{1/3} = \boxed{e^{1/3}}$

16. a. $\frac{d}{dx} \int_{-1}^{x^2} \sqrt{t+1}\, dt = \boxed{2x\sqrt{x^2+1}}$

Check: $\int_{-1}^{x^2} \sqrt{t+1}\, dt = \frac{2}{3}(t+1)^{3/2}\Big|_{-1}^{x^2} = \frac{2}{3}(x^2+1)^{3/2}$

$\frac{d}{dx}\left(\frac{2}{3}(x^2+1)^{3/2}\right) = (x^2+1)^{1/2} \cdot 2x = \boxed{2x\sqrt{x^2+1}}$

b. $\frac{d}{dx} \int_{\pi}^{1/x} \sin t\, dt = \left(\sin \frac{1}{x}\right) \cdot \frac{-1}{x^2} = \boxed{\frac{-\sin\left(\frac{1}{x}\right)}{x^2}}$

Check: $\int_{\pi}^{1/x} \sin t\, dt = -\cos t \Big|_{\pi}^{1/x} = -\left(\cos \frac{1}{x}\right) - 1$

$\frac{d}{dx}\left(-\left(\cos \frac{1}{x}\right) - 1\right) = \left(\sin \frac{1}{x}\right) \cdot \frac{-1}{x^2} = \boxed{\frac{-\sin\left(\frac{1}{x}\right)}{x^2}}$

25. a.

$$\frac{d}{dx}\int_{x^2}^{x^3} \sin^2 t\, dt = [\sin^2(x^3)]\cdot 3x^2 - [\sin^2(x^2)]\,2x$$
$$= \boxed{3x^2\sin^2(x^3) - 2x\sin^2(x^2)}$$

b. $$\frac{d}{dx}\int_{-x}^{x}\frac{1}{1+t}\,dt = \frac{1}{1+x} - \frac{1}{1-x}\cdot -1$$
$$= \frac{1}{1+x} + \frac{1}{1-x} = \boxed{\frac{2}{1-x^2}}$$

27. a. $F(0)=0$, $F(3)=0$, $F(5)=6$, $F(7)=6$, $F(10)=3$

These values represent the signed area under the curve from 0 to x.

b. F is increasing whenever the area is positive, i.e. above the x-axis.

increasing: $(\frac{3}{2}, 6)$, $(\frac{37}{4}, 10)$

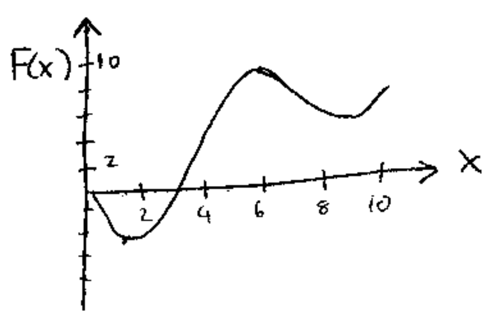
decreasing: $(0, \frac{3}{2})$, $(6, \frac{37}{4})$

c. min at $x=\frac{3}{2}$

max at $x=6$

Check the critical points (i.e. where $f=0$) to find these.

d.



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24. $F(x)=\int_0^x \frac{t-3}{t^2+7}\,dt$

a. F increasing when $F'>0$

$$F'=\frac{x-3}{x^2+7}$$

increasing if $x>3$

decreasing if $x<3$

b. $F''=\frac{d}{dx}\left(\frac{x-3}{x^2+7}\right)=\frac{(x^2+7)-2x(x-3)}{(x^2+7)^2}$

$$=\frac{-x^2+6x+7}{(x^2+7)^2}$$

$F''=0$ when $x=-1,7$

$-$ $+$ $-$ F''

-1 7

Concave up $-1<x<7$

Concave down $x<-1,\ x>7$

c. Critical point where $F'=0$.

$x=3$ is only critical point, and there are no boundaries.

3 is a relative minimum. (from part a, or b/c $F''(3)>0$)

Therefore,

$x=3$ is an absolute minimum.

There are no absolute maxima.