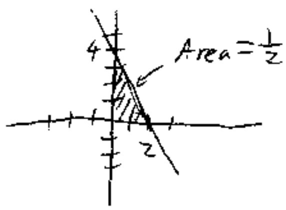
HW #22

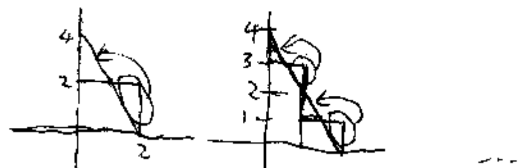
7.1 # 2, 6, 9, 10   7.2 # 8, 9, 10, 14, 18

(2) $f(x) = 4 - 2x \quad [0, 2]$



$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(2)(4)$
$= 4$

Using the midpoints for the rectangle area sum always gives 4 as shown below



$A_1 = 4$

$A_2 = f(.5) + f(1.5)$
$= 3 + 1 = 4$

Using the right end points

$A_1 = 0$ because $f(2) = 0$

$A_2 = 1 \cdot f(1) + 1 \cdot f(2) = 2$

$A_3 = \frac{2}{3} \cdot f\left(\frac{2}{3}\right) + \frac{2}{3} f\left(\frac{4}{3}\right) + \frac{2}{3} f(2) = 8/3$

In general,

$A_n = \frac{2}{n}\left[f\left(\frac{2}{n}\right) + f\left(2 \cdot \frac{2}{n}\right) + f\left(3 \cdot \frac{2}{n}\right) + \dots f\left((n-1)\frac{2}{n}\right) + f\left(n \cdot \frac{2}{n}\right)\right]$

So

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
0	2	8/3	3	3.2	10/3	24/7	3.5	32/9	3.6

(6) $A(x) = 4x - x^2$

$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} = \lim_{h \to 0} \frac{4(x+h) - (x+h)^2 - (4x - x^2)}{h} = \lim_{h \to 0} \frac{4h - 2xh - h^2}{h} = \lim_{h \to 0} 4 - 2x - h = 4 - 2x$ ✓

$A'(x) = 4 - 2x$ so its antiderivative is $4x - 2\left(\frac{x^2}{2}\right) + C = 4x - x^2 + C$

Since $A(0) = 0$, $A(0) = 4(0) - 0^2 + C = 0$, $C = 0$

So $A(x) = 4x - x^2$

$A(2) = 4(2) - 2^2 = 8 - 4 = 4$ ✓

(9) $y = e^x$ Guess: $A(x) = e^x + C$ Check $A'(x) = e^x = f(x)$ ✓ (y)

$A(0) = 0$ so $A(0) = e^0 + C = 0$ so $1 + C = 0$ and $C = -1$

$A(x) = e^x - 1$, $\boxed{A(1) = e - 1}$

(10) $y = \sin x$ Guess: $A(x) = \cos x + C$ Check $A'(x) = -\sin x \neq y$ so guess again

$A(x) = -\cos x + C$ Check $A'(x) = \sin x = y$ ✓

$A(0) = 0$ so $A(0) = -\cos 0 + C = 0$ so $C = 1$

$A(x) = 1 - \cos x \quad A(\pi) = 1 - \cos \pi = 1 - (-1) = 2$