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H.W. # 20 solutions : 6.4 2, 6, 15, 21, 23

2. $f(x) = x^2 - 7$

$f'(x) = 2x$

guess: $x_1 = 3$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{2}{6} = \frac{8}{3}$

$x_3 = \frac{8}{3} - \frac{\left(\frac{8}{3}^2 - 7\right)}{2 \cdot \frac{8}{3}} = \frac{127}{48} \approx 2.6458$

6. $f(x) = x^3 + x - 1$

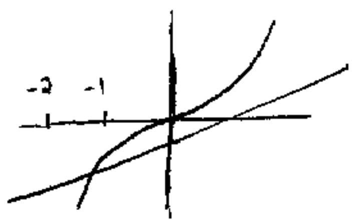
$f'(x) = 3x^2 + 1$

guess: $x_1 = 1$

$x_2 = 1 - \frac{(1)^3 + 1 - 1}{3(1)^2 + 1} = 1 - \frac{1}{4} = \frac{3}{4}$

$x_3 = \frac{3}{4} - \frac{\left(\frac{3}{4}\right)^3 + \frac{3}{4} - 1}{3\left(\frac{3}{4}\right)^2 + 1} \approx .6860$

15. graph:



for Newton's method, use

$f(x) = x^3 - \frac{x}{2} + 1$

guess: $x_1 = -1$

$f'(x) = 3x^2 - \frac{1}{2}$

$$X_2 = -1 - \frac{(-1)^3 - (-\frac{1}{2}) + 1}{3(-1)^2 - \frac{1}{2}} = -1 - \frac{\frac{1}{2}}{\frac{5}{2}} = -\frac{6}{5}$$

$$X_3 = -\frac{6}{5} - \frac{(-\frac{6}{5})^3 - (-\frac{3}{5}) + 1}{3(-\frac{6}{5})^2 - \frac{1}{2}} \approx -1.1665$$

21. absolute minimum occurs at extrema point, so we want point where
$f'(x) = 0$

let $g(x) = f'(x) = x^3 + 2x + 5$

$g'(x) = 3x^2 + 2$

guess: $x_1 = -1$

$X_2 = -1 - \left(\frac{-1-2+5}{5}\right) = -1.4$

$X_3 = -1.4 - \left(\frac{(-1.4)^3 - 2.8 + 5}{3(-1.4)^2 + 2}\right) \approx -1.33$

since local min occurs at about $x = -1.33$ and this is clearly an absolute min. by the shape of the graph, the absolute min can be approximated by $f(-1.33) \approx -4.1$

23. minimize the square of the distance formula: point on parabola has coordinates (x, x^2) so the distance between one such point and $(1, 0)$ is

$$\sqrt{(x-1)^2 + (x^2)^2}$$

so we will minimize $(x-1)^2 + (x^2)^2 = x^4 + x^2 - 2x + 1$

minimum occurs at extremum, so $f'(x) = 0$.

$$4x^3 + 2x - 2 = 0$$

Now use Newton's method to solve

$$g(x) = f'(x) = 4x^3 + 2x - 2 = 0$$

$$g'(x) = 12x^2 + 2$$

guess $x_1 = .5$

$$x_2 = .5 - \frac{4(.5)^3 + 2(.5) - 2}{12(.5)^2 + 2} = .5 - \left(\frac{-.5}{5}\right)$$

$$= .6$$

$$x_3 = .6 - \frac{4(.6)^3 + 2(.6) - 2}{12(.6)^2 + 2} = .6 - \frac{.064}{6.32}$$

$$\approx .5899$$

so this is the x coordinate of the point. The y coordinate is its square, so the point on the parabola closest to $(1, 0)$ is approximately

$$(.5899, .348)$$