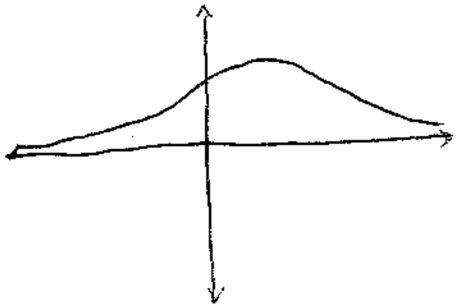
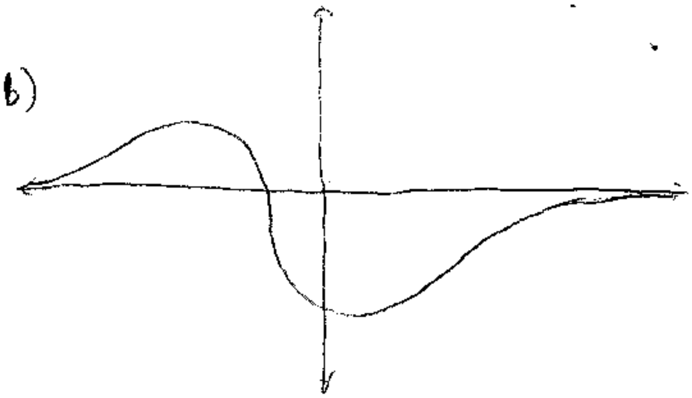
Section 5.2

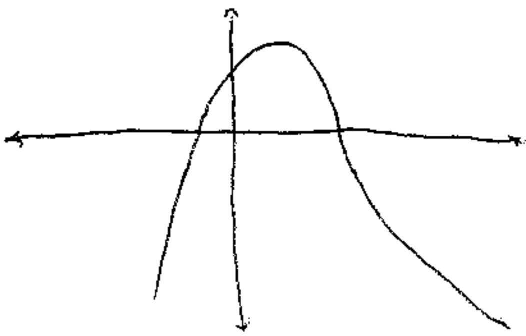
2 (a)



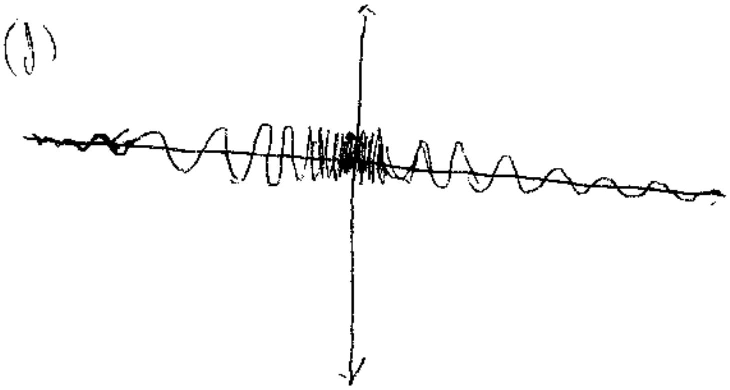
(b)



(c)



(d)



10 (a) $f(x) = \frac{x^2-3}{x^2+1}$ $f'(x) = \frac{(x^2+1)(2x) - (x^2-3)(2x)}{(x^2+1)^2} = \frac{8x}{(x^2+1)^2}$

Stationary point at $x=0$ because $f(0)=0$

No points of nondifferentiability because $f'(x)$ is ~~de~~ continuous and finite for all x.

(b) $f(x) = (x+2)^{1/3}$ $f'(x) = \frac{1}{3}(x+2)^{-2/3}$

No stationary pts because $f'(x) \neq 0$ for all x.

Point of nondifferentiability at $x=-2$ because $f'(x)$ is unbounded at $x=-2$

14 (a) Relative minima of $f(x)$ occur when $f'(x)$ goes from negative to positive

$x=1$

(b) Relative maxima occur when $f'(x)$ goes ~~m~~ from positive to negative.

$x=5$

(c) Inflection pts occur when $f'(x)$ has a horizontal tangent line.

$x = 0, -1, 3$

(19) $f(x) = \sin^2 x \qquad 0 < x < 2\pi$

$f'(x) = 2\sin x \cos x \qquad f'(x)=0$ at $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

First derivative test:

~~At x=0~~ ~~[illegible]~~

$f'(x)<0$ for x slightly less than zero and $f'(x)>0$ for x slightly greater than zero

$x=0$ is a minimum

$f'(x)>0$ for x slightly less than $\pi/2$ and $f'(x)<0$ for x slightly greater than $\pi/2$

$x=\pi/2$ is a maximum

$f'(x)<0$ for x slightly less than π and $f'(x)>0$ for x slightly greater than π

$x=\pi$ is a minimum

$f'(x)>0$ for x slightly less than $\frac{3\pi}{2}$ and $f'(x)<0$ for x slightly greater than $3\pi/2$

$x=\frac{3\pi}{2}$ is a maximum

$f'(x)<0$ for x slightly less than π and $f'(x)>0$ for x slightly greater than π

$x=\pi$ is a minimum

Second derivative test:

$f''(x) = -2\sin^2 x + 2\cos^2 x$

$f''(0) = 2 > 0 \qquad 0$ is a min

$f''(\pi/2) = -2 < 0 \qquad \pi/2$ is a max

$f''(\pi) = 2 > 0 \qquad \pi$ is a min

$f''(\frac{3\pi}{2}) = -2 < 0 \qquad \frac{3\pi}{2}$ is a max

$f''(2\pi) = 2 > 0 \qquad \pi$ is a min

(23) $f(x) = x(x-1)^2 = x(x^2-2x+1) = x^3-2x^2+x$

$f'(x) = 3x^2-4x+1$

Critical points occur where $f'(x)=0$ or $3x^2-4x+1=0$

$(3x-1)(x-1)=0$

$x = \frac{1}{3}, 1 \qquad$ Note: $f''(x) = 6x-4$

$f''(\frac{1}{3}) < 0$ so $\frac{1}{3}$ is a relative max

$f''(1) > 0$ so 1 is a relative min

(32) $f(x) = x^2e^x$

$f'(x) = 2xe^x + x^2e^x$

$f''(x) = 2e^x + 2xe^x + 2xe^x + x^2e^x = (2+4x+x^2)e^x$

Critical points occur where $f'(x)=0$ or $f'(x)$ is undefined.

$2xe^x + x^2e^x = 0 \qquad (2x+x^2)e^x = 0$

Note that $e^x > 0$ for all x

$x = 0, -2$

$f''(0) = 2 > 0 \qquad f(0) = 0$

$f''(-2) = -2e^{-2} < 0 \qquad f(-2) = 4e^{-2}$

$(0,0)$ is a relative min.

$(-2, 4e^{-2})$ is a relative max.

(49) We must find k such that $f'(3) = 0$ and $f''(3) \neq 0$

(a) $f(x) = x^2 + kx^{-1}$

$f'(x) = 2x - kx^{-2}$

$f'(3) = 0 \Rightarrow \quad 0 = 2\cdot 3 - k(3^{-2})$

$0 = 6 - \frac{k}{9}$

$k = 54$

Check that $k=54$ satisfies $f''(3) \neq 0$

$f''(x) = 2 + 54x^{-3} \qquad f''(3) = 2 + \frac{54}{27} \neq 0$ It checks

(b) $f(x) = \dfrac{x}{x^2+k} \qquad f'(x) = \dfrac{x^2+k - x(2x)}{(x^2+k)^2} = \dfrac{-x^2+k}{(x^2+k)^2}$

$f'(3) = 0 \Rightarrow \quad \dfrac{-9+k}{(9+k)^2} = 0 \qquad k = 9$

Check that $k=9$ satisfies $f''(3) \neq 0$

$$f''(x) = \frac{(x^2+k)^2(-2x) - (k-x^2)(x^2+k)(4x)}{(x^2+k)^4} = \frac{(x^2+k)(-2x) - (k-x^2)(4x)}{(x^2+k)^3}$$

$$f''(3) = \frac{(9+9)(-6) - (9-9)(12)}{(9+9)^2} \neq 0$$

$k=9$ satisfies $f''(3) \neq 0$