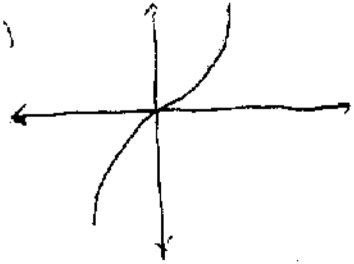
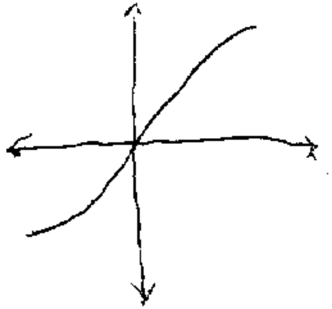
Section 5.1

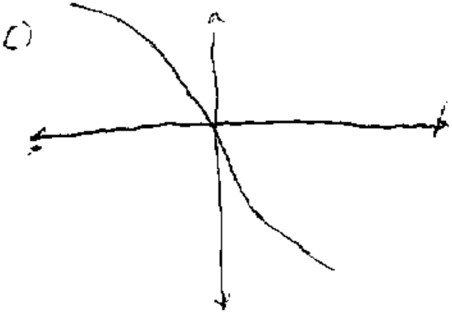
(circled mark) (a)



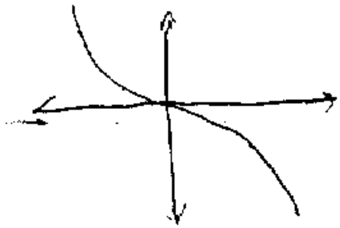
(b)



(c)



(d)



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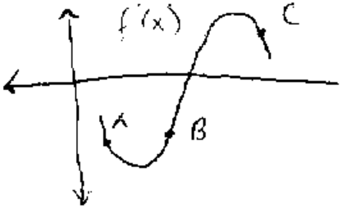

$\frac{dy}{dx}$ is just another name for $f'(x)$, so $\frac{dy}{dx}$ is negative for A and B and positive at C

$\frac{d^2y}{dx^2}$ is the first derivative of $f'(x)$ so you need to see where the graph is increasing or decreasing.

$\frac{d^2y}{dx^2}$ is positive at B and negative at A and C

(9) $f(x) = x^2 - 5x + 6$

$f'(x) = 2x - 5$

$f''(x) = 2$

(a) $f'(x) > 0$ when $2x - 5 > 0$ or $x > \frac{5}{2}$

$f(x)$ increases for $x \in \left(\frac{5}{2}, \infty\right)$

(b) $f'(x) < 0$ when $x < \frac{5}{2}$

$f(x)$ decreases on the interval $\left(-\infty, \frac{5}{2}\right)$

(c) Concave up when $f''(x) > 0$ or $2 > 0$ this is true for all x

concave up for $x \in (-\infty, \infty)$

(d) Concave down when $2 < 0$, i.e. never

Concave down never.

(e) There are no inflection points since it is always concave up.

(14) $f(x) = x^4 - 8x^2 + 16$

$f'(x) = 4x^3 - 16x = x(4x^2 - 16) = 4x(x+2)(x-2)$

$f''(x) = 12x^2 - 16$

Note that stationary points are $x = 0, 2$, and -2

The points when $f''(x) = 0$ are $x = \pm \frac{2}{\sqrt{3}}$

(a) $f'(x) > 0$ for $x \in (-2, 0) \cup (2, \infty)$

(b) $f'(x) < 0$ for $x \in (-\infty, -2) \cup (0, 2)$

(c) $f''(x) > 0$ for $x \in \left(-\infty, \frac{-2}{\sqrt{3}}\right) \cup \left(\frac{2}{\sqrt{3}}, \infty\right)$

(d) $f''(x) < 0$ for $x \in \left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

(e) Inflection points where the concavity changes:

$x = \frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(22)

$f(x) = xe^{x^2}$

$f'(x) = e^{x^2} + 2x^2e^{x^2}$

$f''(x) = 2xe^{x^2} + 4xe^{x^2} + 4x^3e^{x^2} = 6xe^{x^2} + 4x^3e^{x^2}$

Note $f'(x) = (1+2x^2)\,e^{x^2}$

$f''(x) = x(6+4x^2)\,e^{x^2}$

Note that $e^{x^2} > 0$ for all x and $6+4x^2 > 0$ for all x

(a) $f'(x) = (1+2x^2)e^{x^2}$ is always greater than zero

$f(x)$ increases on $(-\infty, \infty)$

(b) $f(x)$ never decreases

(c) Concave up when $x(6+4x^2)e^{x^2} > 0$ which is the same as when $x(6+4x^2) > 0$ or $x > 0$

Concave up on $(0, \infty)$

(d) Concave down on $(-\infty, 0)$

(e) Inflection point at $x=0$ where concavity changes.