

§4.7

#1) a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 8} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x+4)(x-2)} = \lim_{x \rightarrow 2} \frac{x+2}{x+4} = \frac{2}{3}$

$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 8} = \lim_{x \rightarrow 2} \frac{x-2}{x+2} = \frac{2}{3} \checkmark$

b) $\lim_{x \rightarrow \infty} \frac{2x - 5}{3x + 7} = \lim_{x \rightarrow \infty} \frac{2 - 5/x}{3 + 7/x} = \frac{2}{3}$

$\lim_{x \rightarrow \infty} \frac{2x - 5}{3x + 7} = \lim_{x \rightarrow \infty} \frac{2}{3} = \frac{2}{3} \checkmark$

#10) $\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0^+} \frac{\cos x}{2x} = \lim_{x \rightarrow 0^+} \frac{1}{2x} = \infty$

#15) $\lim_{x \rightarrow \infty} \frac{x^{100}}{e^x} = \lim_{x \rightarrow \infty} \frac{100x^{99}}{e^x} = \dots = \lim_{x \rightarrow \infty} \frac{100!}{e^x} = 0$

#27) let $y = \lim_{x \rightarrow 0} (e^x + x)^{1/x}$

$\ln y = \lim_{x \rightarrow 0} \frac{1}{x} \ln(e^x + x) = \lim_{x \rightarrow 0} \frac{\ln(e^x + x)}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{e^x + x} \cdot (e^x + 1)}{1}$
 $= \frac{1}{1} \cdot (1+1) = 2$

$\ln y = 2 \Rightarrow y = \boxed{e^2}$

#60) a) $\lim_{x \rightarrow 0} \frac{x^2 \sin 1/x}{\sin x} = \lim_{x \rightarrow 0} \frac{2x \sin \frac{1}{x} + x^2 \left(\frac{-1}{x^2} \right) \cos \frac{1}{x}}{\cos x}$

$= \lim_{x \rightarrow 0} \frac{2x \sin 1/x - \cos 1/x}{\cos x}$ ← D.N.E. because of $\cos \frac{1}{x}$ term.

b) $\lim_{x \rightarrow 0} \frac{x^2 \sin 1/x}{\sin x} = \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right) \cdot x \sin \frac{1}{x} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = \boxed{0}$