HW # 11 solutions

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Section 4.4

2. $\frac{1}{x^3} \cdot 3x^2 = 3x^{-1} = \frac{3}{x}$

4. $\frac{1}{\sin x} \cdot \cos x = \cot x$

6. $\frac{1}{2+\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2+\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{4\sqrt{x}+2x}$

8. $\frac{1}{x \ln x}$

10. $\frac{x^3}{x} + 3x^2 \ln x = x^2(1 + 3\ln x)$

12. $\frac{1}{2}\left|1+\ln^2 x\right|^{-\frac{1}{2}} (2\ln x)\frac{1}{x} = \frac{\ln x}{x\sqrt{1+\ln^2 x}}$

20. $(e^{-5x^2})(-10x) = -10x e^{-5x^2}$

22. $(e^{\frac{1}{x}})(-\frac{1}{x^2}) = \frac{-e^{\frac{1}{x}}}{x^2}$

24. $(e^x)\cos(e^x)$

30. $\frac{1}{\cos e^x}(-\sin e^x)e^x = -\tan e^x (e^x)$

32. $\frac{dy}{dx} = \frac{1}{x\tan y}\left(x\sec^2 y \frac{dy}{dx} + \tan y\right)$

$$x\tan y \frac{dy}{dx} - x\sec^2 y \frac{dy}{dx} = \tan y$$

$$\frac{dy}{dx} = \frac{\tan y}{x\tan y - x\sec^2 y} = \frac{\frac{\sin y}{\cos y}}{\frac{x}{\cos y}\left(\sin y - \frac{1}{\cos y}\right)}$$

$$= \frac{\sin y}{x(\sin y - \sec y)}$$

40. From formula 14, $f'(x) = -3^{-x}\ln 3$

From log. diff.

$$\frac{dy}{dx} = y\frac{d\ln y}{dx} \Rightarrow \ln y = \ln(3^{-x})$$
$$= -x\ln 3$$
$$\frac{d\ln y}{dx} = -\ln 3$$

so $\frac{dy}{dx} = -(3^{-x})(\ln 3)$ ✓

44. $$\frac{dy}{dx} = y\frac{d\ln y}{dx}$$
$$\ln y = (\sin x)\ln(x)$$
$$\frac{d\ln y}{dx} = \cos x \ln x + \frac{\sin x}{x}$$
$$\frac{dy}{dx} = y\frac{d\ln y}{dx} = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

47. $y = Ae^{2x} + Be^{-4x}$
$y' = 2Ae^{2x} - 4Be^{-4x}$
$y'' = 4Ae^{2x} + 16Be^{-4x}$

$y'' + 2y - 8y$
$= 4Ae^{2x} + 16Be^{-4x}$
$+ 4Ae^{2x} - 8Be^{-4x}$
$- 8Ae^{2x} - 8Be^{-4x}$

$0e^{2x} + 0e^{-4x} = 0$ ✓

54. $\beta = 10 \log (I/I_0)$

$\frac{d\beta}{dI} = \frac{10}{(I/I_0)\ln 10} \cdot \left(\frac{1}{I_0}\right)$ ← constant, comes from chain rule

a. $I/I_0 = 10$, $\frac{d\beta}{dI} = \frac{1}{\ln 10\ (I_0)}$

b. $I/I_0 = 100$, $\frac{d\beta}{dI} = \frac{1}{10 \ln 10\ (I_0)}$

c. $I/I_0 = 1000$, $\frac{d\beta}{dI} = \frac{1}{100 \ln 10\ (I_0)}$

55. $k = k_0 e^{\left(\frac{-q(T-T_0)}{2T_0T}\right)}$

$= k_0 e^{\frac{-qT + qT_0}{2T_0T}}$

$= k_0 e^{-\frac{q}{2T_0} + \frac{qT_0}{2T_0T}} = k_0 e^{\left(\frac{-q}{2T_0} + \frac{q}{2T}\right)}$

↖ constant

$\frac{dk}{dt} = \frac{-q\ k_0\ e^{\left(\frac{-q}{2T_0} + \frac{q}{2T}\right)}}{2T^2} = \frac{-q k_0}{2T^2} e^{\left(\frac{-q(T-T_0)}{2T_0T}\right)}$