HW #7: §3.5 #4, 6, 8, 9, 10, 18, 24, 42, 48, 64, 66

④ $f(x) = \dfrac{1}{(x^5 - x + 1)^9}$ $\quad f'(x) = -9(x^5 - x + 1)^{-10} \cdot (5x^4 - 1)$

$$= \boxed{\frac{-9(5x^4 - 1)}{(x^5 - x + 1)^{10}}}$$

⑥ $f(x) = \sqrt{x^3 - 2x + 5}$ $\quad f'(x) = \frac{1}{2}(x^3 - 2x + 5)^{-1/2} \cdot (3x^2 - 2)$

$$= \boxed{\frac{3x^2 - 2}{2\sqrt{x^3 - 2x + 5}}}$$

⑧ $f(x) = \sin(x^3)$ $\quad f'(x) = \cos(x^3) \cdot 3x^2$

↕ switch

$$= \boxed{3x^2 \cos(x^3)}$$

⑨ $f(x) = \sin^3(x)$ $\quad f'(x) = (3\sin^2(x)) \cdot (\cos x)$

$$= \boxed{3\sin^2 x \cos x}$$

⑩ $f(x) = \cos^2(3\sqrt{x})$

$f'(x) = 2\cos(3\sqrt{x}) \cdot (-\sin(3\sqrt{x})) \cdot (3 \cdot \frac{1}{2} x^{-1/2})$

$$= \boxed{\frac{-3\sin(3\sqrt{x})\cos(3\sqrt{x})}{\sqrt{x}} = -\frac{3}{2}\frac{\sin(6\sqrt{x})}{\sqrt{x}}}$$

(18) $f(x) = \cos^3 \frac{x}{x+1}$

$$f'(x) = 3\cos^2 \frac{x}{x+1} \cdot \left(-\sin \frac{x}{x+1}\right) \cdot \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$= \boxed{\frac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}}$$

(24) $f(x) = \frac{x}{\sqrt{1-x^2}}$

$$f'(x) = \frac{\sqrt{1-x^2}\cdot 1 - x\left(\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\right)}{1-x^2} = \frac{\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2} = \frac{1}{(1-x^2)^{3/2}} = \boxed{(1-x^2)^{-3/2}}$$

(42) $y = x\tan\left(\frac{1}{x}\right)$ $\quad \frac{dy}{dx} = \tan\frac{1}{x}\cdot 1 + x\cdot\left(\sec^2\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$

$$= \tan\frac{1}{x} - \frac{1}{x}\sec^2\frac{1}{x}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \sec^2\left(\frac{1}{x}\right)\cdot\left(\frac{-1}{x^2}\right) - \left[\frac{-1}{x^2}\sec^2\left(\frac{1}{x}\right) + \frac{1}{x}\cdot 2\sec\frac{1}{x}\cdot\left(\sec\frac{1}{x}\tan\frac{1}{x}\right)\cdot\frac{-1}{x^2}\right]$$

$$= \frac{-\sec^2\frac{1}{x}}{x^2} + \frac{\sec^2\frac{1}{x}}{x^2} + \frac{2}{x^3}\sec^2\frac{1}{x}\tan\frac{1}{x}$$

$$= \boxed{\frac{2}{x^3}\sec^2\frac{1}{x}\cdot\tan\frac{1}{x}}$$

(48) $y = (x - 1/x)^3$ Eqn. of tangent line @ $x = 2$?

$$\frac{dy}{dx} = 3\left(x - \frac{1}{x}\right)^2\left(1 + \frac{1}{x^2}\right)$$

@ $x=2$: $\frac{dy}{dx}(x=2) = 3\left(2 - \frac{1}{2}\right)^2 \cdot \left(1 + \frac{1}{4}\right) = \frac{135}{16}$

@ $x=2$: ① $y = \left(2 - \frac{1}{2}\right)^3 = \frac{27}{8}$

② $\frac{dy}{dx} = \frac{135}{16}$

Point-slope form:

$$y - y_0 = \frac{dy}{dx}(x_0) \cdot (x - x_0)$$

$$\boxed{y - \frac{27}{8} = \frac{135}{16}(x - 2)}$$

(64) $F(x) = f(g(x))$, $G(x) = g(f(x))$

(a) $F'(-1) = f'(g(-1)) \cdot g'(-1) = f'(2) \cdot (-3) = 4(-3) = \boxed{12}$

(b) $G'(-1) = g'(f(-1)) \cdot f'(-1) = g'(2) \cdot 3 = (-5) \cdot 3 = \boxed{-15}$

(66) Given $f'(x) = \sqrt{3x+4}$

$g(x) = x^2 - 1 \longrightarrow g'(x) = 2x$

$F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x) = \sqrt{3 \cdot g(x) + 4} \cdot 2x$

$= 2x\sqrt{3(x^2-1)+4}$

$= \boxed{2x\sqrt{3x^2+1}}$