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Homework # 3 solutions

3.1 2, 4, 6, 8, 9

3.2 2, 12, 18

3.1

#2 a) avg. rate of change $= \frac{\Delta y}{\Delta x} = \frac{2^3 - 1^3}{1} = 7$

b) $\lim_{x_1 \to 1} \frac{(x_1)^3 - 1}{x - 1} = \lim_{x_1 \to 1} \frac{(x_1 - 1)(x^2 + x_1 + 1)}{(x_1 - 1)}$

$= \lim_{x_1 \to 1} x^2 + x_1 + 1$

$= 1 + 1 + 1 = 3$

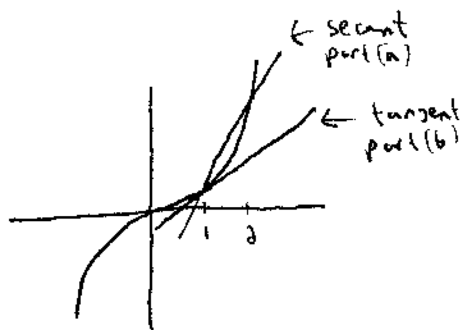
c) $\lim_{x_1 \to x_0} \frac{(x_1)^3 - (x_0)^3}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{(y_1 - x_0)(x_1^2 + x_1 x_0 + x_0^2)}{(x_1 - x_0)}$

$= \lim_{y_1 \to x_0} (x_1^2 + x_1 x_0 + x_0^2)$

$= x_0^2 + x_0^2 + x_0^2$

$= 3x_0^2$

d)



#4 a) avg. $= \frac{\Delta y}{\Delta x} = \frac{(\frac{1}{2})^2 - (\frac{1}{1})^2}{1} = -\frac{3}{4}$

b) $\lim_{x_1 \to 1} \frac{\frac{1}{x_1^2} - 1}{x_1 - 1} = \lim_{x_1 \to 1} \frac{\frac{1 - x_1^2}{x^2}}{x_1 - 1} = \lim_{x_1 \to 1} -\left(\frac{1 + x_1}{x_1}\right)$

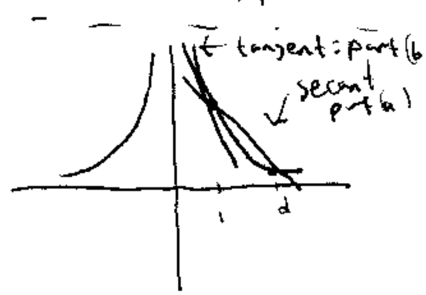
$= -2$

(c) $\lim_{x_1 \to x_0} \frac{\frac{1}{x_1^2} - \frac{1}{x_0^2}}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{\frac{x_0^2 - x_1^2}{x_1^2 x_0^2}}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{(x_0 - x_1)(x_0 + x_1)}{(x_1^2 x_0^2)(x_1 - x_0)}$

$= \lim_{x_1 \to x_0} -\frac{x_0 + x_1}{x_1^2 x_0^2}$

$= -\frac{2x_0}{x_0^4} = -2x_0^{-3}$

d)



6

a) $$\lim_{x_1 \to x_0} \frac{x_1^2 + 3x_1 + 2 - (x_0^2 + 3x_0 + 2)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{x_1^2 - x_0^2 + 3x_1 - 3x_0 + 2 - 2}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{(x_1 + x_0)(x_1 - x_0) + 3(x_1 - x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} x_1 + x_0 + 3 = 2x_0 + 3$$

b) $2(x_0) + 3 = 2(2) + 3 = 7$

8

a) $$\lim_{x_1 \to x_0} \frac{\frac{1}{\sqrt{x_1}} - \frac{1}{\sqrt{x_0}}}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{\frac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0 x_1}}}{x_1 - x_0}$$

multiply by $\frac{\sqrt{x_0} + \sqrt{x_1}}{\sqrt{x_0} + \sqrt{x_1}}$

$$\Rightarrow = \lim_{x_1 \to x_0} \frac{x_0 - x_1}{(x_1 - x_0)(\sqrt{x_0 x_1})(\sqrt{x_0} + \sqrt{x_1})}$$

$$= \lim_{x_1 \to x_0} -\frac{1}{(\sqrt{x_0 x_1})(\sqrt{x_0} + \sqrt{x_1})}$$

$$= -\frac{1}{\sqrt{x_0^2}\, 2\sqrt{x_0}}$$

$$= -\frac{1}{2} x_0^{-3/2}$$

b) $x_0 = 4$, $-\frac{1}{2}(4)^{-\frac{3}{2}} = -\frac{1}{2} \cdot \frac{1}{8} = -\frac{1}{16}$

9. a) instantaneous velocity = slope of tangent at $t = 10$ s.

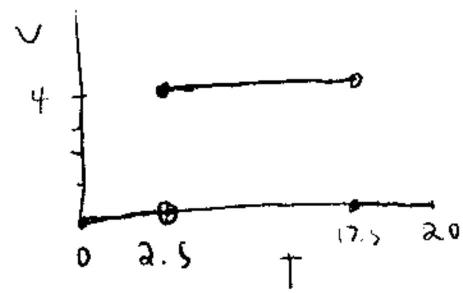
since between $t=10$ and $t=15$, we have a straight line, the slope of the tangent = slope of the line

= slope of secant that connects $t=10$ and $t=15$

= avg. velocity between $t=10$ and $t=15$

$$= \frac{50-30}{15-10} = \frac{20}{5} = 4 \text{ m/s}$$

b)



Since between 2.5 and 17.5 V constant at 4

otherwise, v constant at 0.

Section 3.1

10. (intuitively).

(a) Since $d(0) = d(3) = 10$, $\boxed{\text{average velocity is } 0}$.

(b) The instaneous velocity is zero when the object turns around; it looks like this occurs at

$$\boxed{t = 0, 2, 4, 8}$$

(c) These will be the values of t where the distance function has the greatest slope. This looks like it happens either at $t = 1$ (where velocity is positive) or $t = 3$ (where velocity is negative). Thus, at $t = 1$ the object achieves its maximum velocity, and at $t = 3$ its minimum velocity.

(d) The segment from $t = 2$ to $t = 4$ is approximately a straight line whose slope is $\frac{5-15}{4-2} = -5$
$\Rightarrow$ instaneous velocity at $t = 3$ is approximately $\boxed{-5}$.