Section 1.2

4. (a) $f(x) = \frac{1}{5x+7}$. The domain is everywhere that the denominator is nonzero. $5x+7=0$ only when $x = -\frac{7}{5}$ $\Rightarrow$ domain of f is $\{x \in \mathbb{R} \mid x \neq -\frac{7}{5}\}$ which can also be denoted $\mathbb{R} \setminus \{-\frac{7}{5}\}$ or

$$(-\infty, -\tfrac{7}{5}) \cup (-\tfrac{7}{5}, \infty).$$

(b) $h(x) = \sqrt{x-3x^2}$ is defined whenever $x - 3x^2 \geq 0$

$\Rightarrow$ $x(1-3x) \geq 0$, which happens when

$x \geq 0$ and $1-3x \geq 0$ -or- $x \leq 0$ and $1-3x \leq 0$.

$1-3x \geq 0 \Rightarrow 1 \geq 3x$

$\Rightarrow x \leq \frac{1}{3}$

but this never happens, since $1-3x > 0$ for all $x \leq 0$.

Thus, we see that the domain of h is $\{x \in \mathbb{R} \mid 0 \leq x \leq \frac{1}{3}\}$, which can also be written as $[0, 3]$.

(c) $G(x) = \sqrt{\frac{x^2-4}{x-4}}$ First of all, the denominator can't equal zero $\Rightarrow$ $x \neq 4$.

Also, since we're taking the square root, we need

$$\frac{x^2-4}{x-4} \geq 0 \Rightarrow x^2-4 \geq 0 \text{ and } x-4 > 0, \text{ -or- }$$
$$x^2-4 \leq 0 \text{ and } x-4 < 0.$$

In the first case, note that $x-4>0 \Rightarrow x>4$.

Also, $x>4 \Rightarrow x^2-4 > 4^2-4 = 12 > 0$.

Thus, all $x>4$ are allowable (in the domain).

Now for the second case: $x^2-4 \leq 0$ only when $x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$.

Note that for all such x, $x-4 \leq 2-4 = -2 < 0$.

Thus, they're all in the domain.

So, the domain is $\{x \in \mathbb{R} \mid -2 \leq x \leq 2, \text{ or } x > 4\}$, which is also denoted $[-2,2] \cup (4,\infty)$.

(d) $f(x) = \frac{x^2-1}{x+1}$ In this function, all we have to worry about is the keeping the denominator nonzero. $x+1=0 \implies x=-1$

Everything else will be well-defined.

Thus, the domain of f is $\{x \in \mathbb{R} \mid x \neq -1\}$, which can also be written as $\mathbb{R} \setminus \{-1\}$ or $(-\infty, -1) \cup (-1, \infty)$.

e) $h(x) = \frac{3}{2-\cos x}$. Once again, we just have to check when the denominator is zero.

Well, $2 - \cos x = 0 \implies \cos x = 2$.

But this can never happen, since the biggest cosine ever gets is 1. Thus, h is defined for all x; that is, the domain of h is $\mathbb{R}$.