Math 1a Exam 2 Review Problems - Fall 1999

1) A function $f(x)$ and its first and second derivatives are given below.

$$f(x) = \frac{x^2 + 4x + 3}{x(x+4)}$$

$$f'(x) = \frac{-6x - 12}{x^2(x+4)^2}$$

$$f''(x) = \frac{6(3x^2 + 12x + 16)}{x^3(x+4)^3}$$

a) Identify any x or y intercepts for the graph of this function.

b) Find any critical points and determine whether they are relative maxima or minima or neither.

c) Find any points of inflection.

d) Find any vertical asymptotes for this graph.

e) Find any horizontal asymptotes for this graph.

f) Give a rough sketch of the graph of this function, illustrating all of the features you found in parts a) through e).

2) a) Find $\lim_{x\to 1}\left(\frac{x^4 - 5x^2 + 3x + 1}{x^3 - 2x + 1}\right)$

b) Find $\lim_{x\to 0}\frac{\sin x - x}{(\cos x - 1)x}$

c) Find $\lim_{x\to \infty}\left(\frac{x+4}{x-1}\right)^x$

3) a) Isaac is using Newton's Method to find a root for the function $f(x) = x^4 - 2$.
His first approximation is $x_0 = 1$. What will be his next approximation?

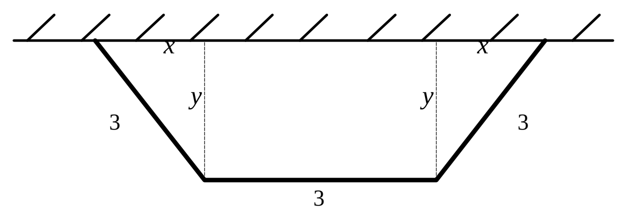
b) Calculate the first two iterates of Newton's Method (that is, x_1 and x_2) for finding the roots of the function $f(x) = x^3 - 7x - 5$ if your initial guess at a root was $x_0 = 3$.

4) Graph the function $f(x) = xe^{-2x}$, labeling on your graph all significant features such as maxima, minima, intercepts, points of inflection, and asymptotes.

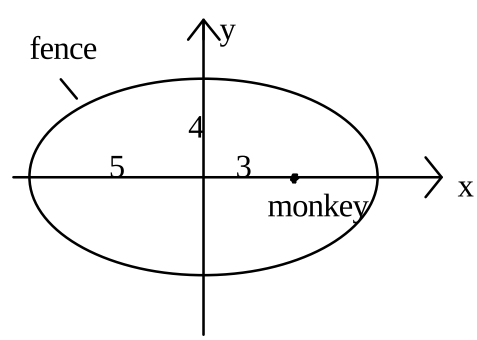
5) Let $g(x) = x^4 - 2x^2 - x + 5$. Explain why there is at least one point $x = c$ in the interval [0,2] at which the slope of the tangent line to the graph of g is equal to 3.

6) Otto is determined to construct a playpen for Luke, his pet guinea pig. Being a kindly fellow, he wants Luke to have the most area in which to frolic about. The boundary of the playpen will consist of the wall of the house on one side and three straight pieces of length 3 feet each, with the middle piece parallel to the wall of the house, as shown in the figure below. How should Otto build the fence?

[You may express your answer either by giving the lengths x and y <u>or</u> by giving the angle q, as shown.]



7) A monkey habitat in a zoo is enclosed by a fence in elliptic shape (if viewed from above), of the dimensions shown below. A small, tired monkey is resting at the marked spot (on the ground). Where along the fence should you stand to be as close to the monkey as possible? (Hint: To simplify the computations, minimize the square of the distance.)



The equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Mark the solution(s) on the sketch, and find the x and y coordinates of the solution(s).

8) Graph the function $f(x) = 3x^2 + 2x^3$, indicating any critical points, relative maxima, relative minima, intercepts, and points of inflection.

9) An unidentified flying object (UFO) is moving in a straight line with acceleration of $a(t) = -t^2$ miles per hour per hour, where t is the time in hours. At $t = 0$, the UFO is directly above Boston and is flying due east at an unknown velocity v_0. In the course of its flight, the UFO gets to a maximum distance of 4 miles east of Boston.

a) Find an expression for the velocity for $t \geq 0$.

b) Find an expression for the position $s(t)$ for $t \geq 0$, where $s = 0$ corresponds to Boston.

c) Find the initial velocity v_0.

10) a) Sketch the graph of $y = x^{2/3} - 2x^{1/3}$ over the interval [-1, 8] showing all significant features such as intercepts, critical points, maxima, minima, points of inflections, asymptotes, etc.

b) Does this function satisfy the conditions of the Mean Value Theorem over this interval? If so, why so? If not, why not?

11) An isosceles triangle has base 6 inches and height 12 inches. Find the maximum possible area of a rectangle that can be placed inside the triangle with one side on the base of the triangle.

12) a) Sketch the graph of the function

$$f(x) = -2x^4 + 3x^2 + 11/8$$

b) Show that Newton's Method fails if we choose $x_0 = 1/2$.

c) Sketch what is happening on the graph.

13) The management of a neighborhood health clinic is reviewing its monthly budget. Each month, it can spend $100,000 to pay the doctors and nurses it employs. The monthly salaries are $10,000 for doctors and $4,000 for nurses. It is estimated that the number of patients which can be seen in this clinic each month is proportional to the product of the number of doctors and nurses working at the clinic. If the goal is to maximize the number of patients who can be seen, how many doctors and nurses should be hired? (Fractional answers are acceptable, since people can be hired part-time.)

14) What are the dimensions of the cylindrical aluminum can that holds 40 cm^3 of oil and which uses the least amount of aluminum?

15) Air France is organizing a special trip around the world on the Concorde. Only one such plane will be available, with a capacity of 120 passengers. Market research observe: When the price for the trip is set at FF 100,000, 60 people will sign up, and for every 5% they lower the price from that level, 10 more people will be interested. How should they set the price to maximize their revenue?

16) An object moves along a straight line, with velocity $v(t) = t^2$. How far does the object move between $t = 1$ and $t = 2$?

17) At $t = 0$, a bagel is travelling in a straight line with a velocity of 25 feet per second. Starting at $t = 0$, the bagel undergoes an acceleration of $a(t) = -5 - 4t$ feet per second per second, where t is the time in seconds.

a) Find an expression for the velocity for $t \geq 0$.

b) At what time(s) is the bagel not moving?

c) How far does the bagel travel between $t = 0$ and $t = 2$?

18)Calculate the following derivatives:

a) $f(x) = \tan^{-1}(\frac{x-1}{x+1})$

b) $h'(-1)$ if $h(x) = \dfrac{x^2 \ln(3x+4)}{x^3+2}$.

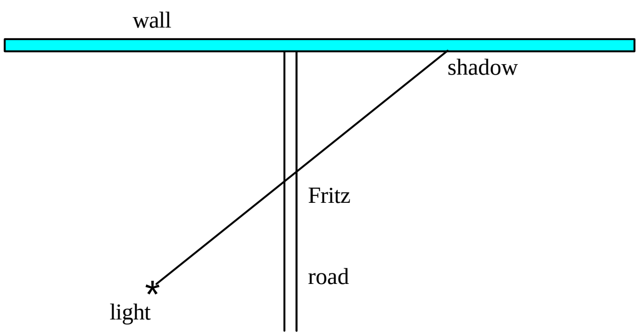
c) $g'(t)$ if $g(t) = p\, t^3 + 3^{p t} + p^3 t$.

d) $\dfrac{dy}{dx}$ at the point (0,-1) for the relation $e^{(y^2+y)} = xy + 1$.

19) A conical paper cup 8 inches across the top and 6 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep?

20) An airplane is maintaining a constant altitude of 5 miles above sea level and is traveling at 507 miles per hour due west of LaGuardia Airport. When the airplane passes over a building 12 miles west of the airport, how fast is the angle between the airplane and the airport changing? Please include units in your answer.

21) A road runs at a right angle to a wall. There is a lamp on the ground 5 meters from the road and 10 meters from the wall. Fritz the Cat is walking on the road towards the wall. Find the rate at which Fritz's shadow is moving along the wall at the instant when he is 5 meters away from the wall, moving at 1 meter per second.



22) A tanker accident has spilled oil in Pristine Bay. Oil-eating bacteria are gobbling up 5 cubic feet of oil per hour. The oil slick has the form of a circular disk of uniform thickness. When the radius of the disk is 500 feet, the thickness of the slick is 0.01 feet and is decreasing at a rate of 0.001 feet per hour.

a) At what rate is the area of the slick changing at this time? ______________________

b) Is the area of the slick increasing or decreasing then?__________________________

23) A rope is fastened to the ground at a distance of eight feet from a vertical wall. A monkey weighing 37 pounds climbs the wall holding onto the loose end of the rope, holding it taut. At which rate is the length of the rope (between the monkey's hand and where it is fastened to the ground) increasing when the monkey is six feet up the wall, climbing at a rate of four feet per second?

24) A spherical ball is expanding. If the radius is increasing at the rate of 2 inches per minute, at what rate is the volume increasing when the radius is 5 inches?

25) Two ships, one heading west and the other east, approach each other on parallel courses 8 nautical miles apart. Given that each ship is cruising at 20 nautical miles per hour, at what rate is the distance between them diminishing when they are 10 nautical miles apart?