

Math 1a - Exam 1 Review Problems - Fall 99

1) Find the following derivatives:

a) $g(t) = \tan t (2+t^3)^5$

b) $f(x) = \tan \sqrt{\frac{x-1}{x+1}}$

c) $f(x) = \cos^2\left(\frac{3}{x}\right)$

d) Find $\frac{dy}{dx}$ if $y = \cos^4(x^5)$

e) Find $f'(x)$ if $f(x) = \sin^2 3x + \cos^2 3x$

f) Find $f'(1)$ for $f(x) = \frac{3}{\sqrt[3]{x}} - x - \frac{1}{x}$

g) Find $\frac{dy}{dx}$ for the relation $x^2 + xy + 2y = 1$

2) Suppose that all we know about a continuous, differentiable function $f(x)$ is the following numerical data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	20	25	27	21	15

Give estimates for $f'(0.6)$ and $f''(0.6)$.

3) Consider the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

a) What is $f'(x)$ for $x \neq 0$?

b) What is $f'(0)$?

4) Which of the following expressions represent the derivative of f at $x = a$?

a) $\lim_{h \rightarrow 0} \frac{f(a+2h) - f(a)}{2h}$

b) $\lim_{x \rightarrow a} \frac{f(x)}{x}$

c) $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h}$

d) $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

e) $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

5) (a) Use the definition of the derivative to compute $f'(0)$ for $f(x) = \frac{1}{2x+1}$.

(b) Use the definition of the derivative to show that if $f(x) = \frac{1}{x^2}$, then $f'(x) = -\frac{2}{x^3}$.

6) Find the equations for two lines through the origin that are tangent to the curve with equation:

$$x^2 - 4x + y^2 + 3 = 0.$$

7) Consider the parabola $y = x^2$. At what point on this parabola is the tangent line parallel to the line connecting the points (1,1) and (3,9) on this parabola?

8) An object travels on a straight line. Its position on that line is given by $s(t) = \frac{2}{3}t^3 - 7t^2 + 20t + 8$, starting at $t = 0$. When its velocity is positive, it is moving to the right. When its velocity is negative, it is moving to the left.

- a) Find a formula for its velocity at time t .
- b) Find a formula for its acceleration at time t .
- c) When is it moving to the right?
- d) When is its velocity decreasing?
- e) When does the object change direction?

9) Suppose that $f(x)$ is a differentiable function whose derivative satisfies $f'(x) = (x^2 - 1)^{-1/2}$. Find

$$\frac{d}{dx} [f(\sec(x))] \text{ for } x \in (0, \frac{\pi}{2}).$$

10) Find $\frac{dy}{dx}$ at the point (0,-1) for the relation $\sec(y^2 + y) = xy + 1$.

11) a) Use the idea of linear approximation for the function $f(x) = \sqrt{x}$ and fact that $(2.5)^2 = 6.25$ to approximate $\sqrt{6}$.

b) Is this approximation less than or greater than the actual value of $\sqrt{6}$? Why?