Review Problems for Math 1a Midterm Exam #2 - Fall 1998

1) a) Find $\lim_{x\to 0}\frac{\sin x - x}{(\cos x - 1)x}$ b) Find $\lim_{x\to\infty}\left(\frac{x+4}{x-1}\right)^x$

2) Calculate the first two iterates of Newton's Method (that is, x_1 and x_2) for finding the roots of the function
$f(x) = x^3 - 7x - 5$ if your initial guess at a root was $x_0 = 3$.

3) Graph the function $f(x) = 3x^2 + 2x^3$, indicating any critical points, relative maxima, relative minima, intercepts, and points of inflection.

4) a) Graph the function $f(x) = \frac{x-1}{x^2-2x}$, indicating any critical points, relative maxima, relative minima, intercepts, points of inflection, and asymptotes.

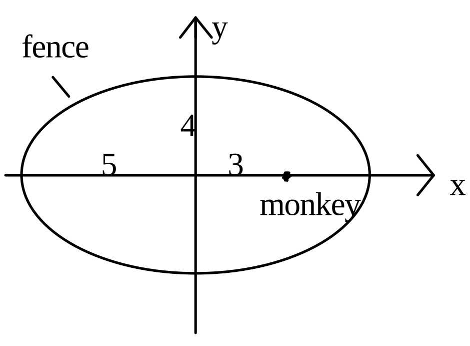
b) Graph the function $f(x) = \frac{x^2-2x}{x-1}$, indicating any critical points, relative maxima, relative minima, intercepts, points of inflection, and asymptotes.

5) a) Sketch the graph of $y = x^{2/3} - 2x^{1/3}$ over the interval [-1, 8] showing all significant features such as intercepts, critical points, maxima, minima, points of inflections, asymptotes, etc.

b) Does this function satisfy the conditions of the Mean Value Theorem over this interval? If so, why so? If not, why not?

6) Bueya is building a cat box for Louis the Cat. The box will have a square bottom, rectangular sides, and an open top. If Bueya is willing to buy 12 square feet of material to use in building the box, how should she build the box so that it contains the most volume and what will that volume be?

7) A monkey habitat in a zoo is enclosed by a fence in elliptic shape (if viewed from above), of the dimensions shown below. A small, tired monkey is resting at the marked spot (on the ground). Where along the fence should you stand to be as close to the monkey as possible? (Hint: To simplify the computations, minimize the square of the distance.)



The equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Mark the solution(s) on the sketch at left, and find the x and y coordinates of the solution(s).

8) An isosceles triangle has base 6 inches and height 12 inches. Find the maximum possible area of a rectangle that can be placed inside the triangle with one side on the base of the triangle.

9) The management of a neighborhood health clinic is reviewing its monthly budget. Each month, it can spend $100,000 to pay the doctors and nurses it employs. The monthly salaries are $10,000 for doctors and $4,000 for nurses. It is estimated that the number of patients which can be seen in this clinic each month is proportional to the product of the number of doctors and nurses working at the clinic. If the goal is to maximize the number of patients who can be seen, how many doctors and nurses should be hired? (Fractional answers are acceptable, since people can be hired part-time.)

10) A manufacturing plant has a capacity of 25 articles per week. Experience has shown that n articles can be sold at a price of p dollars each where p = 110 - 2n and the cost of producing n articles is known to be 600 + 10n + n^2 dollars. How many articles should be made each week to give the largest profit?

11) Find the point on the parabola $y = x^2$ that is closest to the point (4, !/2).

12) Air France is organizing a special trip around the world on the Concorde. Only one such plane will be available, with a capacity of 120 passengers. Market research observe: When the price for the trip is set at FF 100,000, 60 people will sign up, and for every 5% they lower the price from that level, 10 more people will be interested. How should they set the price to maximize their revenue?

13) An architect wants to design a window in the shape of a rectangle capped by a semicircle. If the perimeter of the window is constrained to be 24 feet, what dimensions should the architect choose for the window in order to admit the greatest amount of light?

14) What are the dimensions of the cylindrical aluminum can that holds 40 cm^3 of oil and which uses the least amount of aluminum?

15) An object moves along a straight line, with velocity $v(t) = t^2$.
How far does the object move between $t = 1$ and $t = 2$?

16) a) Sketch the graph of $f(x) = |2x + 1|$. Label x and y intercepts.

b) Compute $\int_{-1}^{2} |2x+1|\, dx$

17) a) Find $I = \int_{1}^{100} \frac{dx}{x^2}$

b) Write down the Riemann Sum R for the integral $\int_{1}^{100} \frac{dx}{x^2}$ with 99 subintervals, using right endpoints. Write your answer as a sum giving at least the first two and the last two summands (and dots in between).

c) Which is larger: Your answer I in part a) or the quantity R in part b)? Explain your answer with a sketch.

d) Is $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{99^2}+\frac{1}{100^2}$ more or less than 2? Justify your answer.