Name: _____ ID#: _____

Midterm I

Math 1a
Introduction to Calculus

27 October 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

Please check your section:

☐	1.0	MWF10	Tatyana Chmutova	☐	4.0	TΘ10	Dawei Chen
☐	1.1	MWF10	Matthew Leingang	☐	4.1	TΘ10	Jerrel Mast
☐	2.0	MWF11	Ethan Cotterill	☐	4.2	TΘ10	Chun-Chun Wu
☐	3.0	MWF12	Matt Bainbridge	☐	5.0	TΘ11:30	Derek Bruff
				☐	5.1	TΘ11:30	Sonal Jain

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—Handbook for Students

Problem Number	Possible Points	Points Earned
1	12	
2	10	
3	10	
4	15	
5	10	
6	8	
7	15	
8	10	
9	10	
Total	100	

1 1

1. (12 Points) Find the following limits.

(i) $\lim_{x \to 1} \dfrac{x^{19} + x - 1}{\arctan(x)}$

(ii) $\lim_{x \to \infty} \left(\sqrt{x^4 + 19} - x^2 \right)$

1

1

(iii) $\lim_{x \to 19} \dfrac{x^3 - 19x^2 - x + 19}{x - 19}$

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2 2

2. (10 Points) We will show that

$$\lim_{x\to\infty} \frac{\sin(x^{19})}{\log x}$$

(a) (7 points) Find two functions f and g such that

$$f(x) \leq \frac{\sin(x^{19})}{\log x} \leq g(x)$$

for all $x > 0$ and

$$\lim_{x\to\infty} f(x) = 0 \qquad\qquad \lim_{x\to\infty} g(x) = 0.$$

Justify your answers.

(b) (3 points) Finish up the proof with two magic words.

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3 3

3. (10 Points) Define the *signum function*

$$\operatorname{sgn}(x) = \begin{cases} 0 & \text{if } x = 0; \\ \dfrac{|x|}{x} & \text{if } x \neq 0. \end{cases}$$

(i) (2 points) What is $\operatorname{sgn}(2)$? $\operatorname{sgn}(-5)$?

(ii) (8 points) Where is sgn continuous? Use limits to justify your answer.

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4 4

4. (15 Points) Determine the vertical and horizontal asymptotes for each of the following functions. For each vertical asymptote, also determine the left-hand and right-hand limits at that asymptote. (For example, if $x = 4$ is a vertical asymptote for f, find $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$.)

(a) $f(x) = \dfrac{5x^2 - 45}{2x^2 - 7x + 3}$

(b) $f(x) = \dfrac{\sqrt{x^2 + 1}}{3x - 1}$

4 4

(c) $f(x) = e^{2x} \cos x$

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5 5

5. (10 Points) Let $f(x) = \sqrt{2x}$. Compute $f'(x)$ *using the definition of derivative.*

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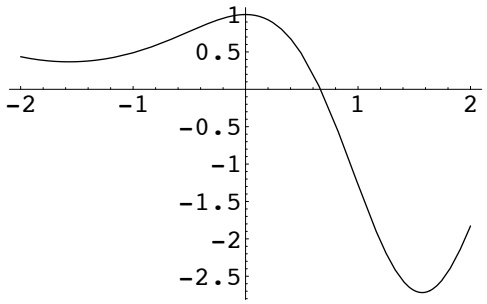
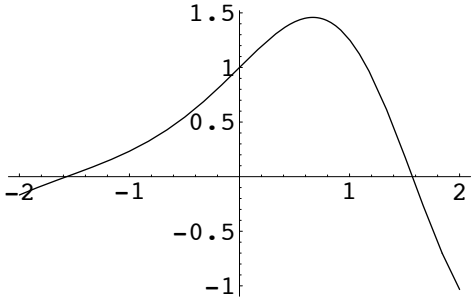
6 6

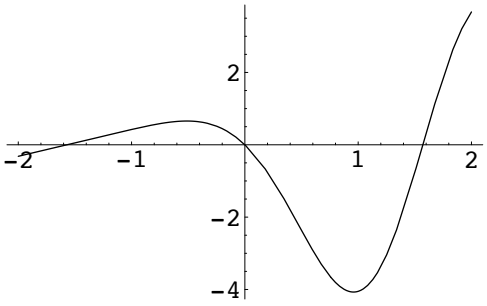
6. (8 Points) Below are the graphs of f and its derivatives f', f'', and f'''. In the blanks below the graphs, write the letter of the graph that corresponds to each of the functions f, f', f'', and f'''.



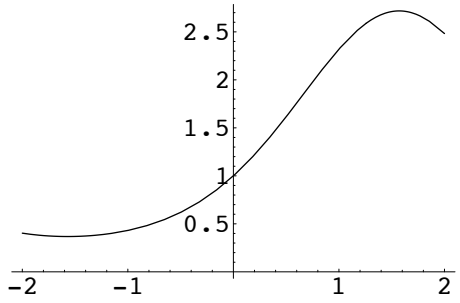
(A)



(B)



(C)



(D)

_____ f

_____ f'

_____ f''

_____ f'''

_____

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7 7

7. (15 Points) Take the following derivatives.

(i) $\dfrac{d}{dx}\left(x^{17}+3x^4+2x-1\right)$

(ii) $\dfrac{d}{dx}\left(\dfrac{x}{1+x^2}\right)$

7

7

(iii) $\dfrac{d}{dx}\left(e^x\left(7x^2+e^x\right)\right)$

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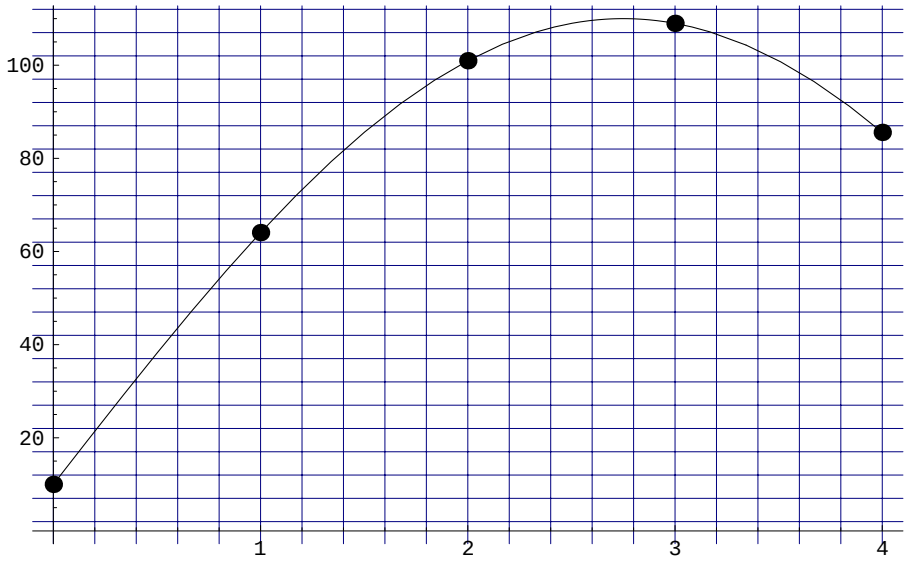
8 8

8. (10 Points) The profits of a small company for each of the first five years of its operation are given in the following table.

Year	Profit in \$1000s
2000	10
2001	64
2002	101
2003	109
2004	86

The company's accountant plotted points representing the profit as a function of years since 2000 and joined the points by a smooth curve. This graph is shown below.



(i) Find the slope of the secant line between the points $(2, 101)$ and $(4, 86)$. What does this slope represent in terms of profit?

(ii) Use the accountant's graph to estimate the rate at which profits were changing in 2002. Clearly describe the method you use to do so.

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9 9

9. (10 Points) Ferdbert Freshman decides to go on a hike. On a Saturday morning he leaves at 8:00AM and climbs Mt. Pennypacker, arriving at 5:00PM. He spends the night at the top of the mountain. The next day, he climbs down the mountain, arriving back at his dorm room at 5:00PM.

Upon Ferdbert's return, his roommate Egbert (taking a break from studying his calculus) says, "Did you know that at some time today you were at the exact same elevation as you were 24 hours before?" "That's impossible," says Ferdbert. "I slept late this morning and didn't start walking until 10:00AM!"

Who's right, and why?

Hint. Let f be the function which is Ferdbert's elevation at time t on Saturday, and g Ferdbert's elevation at time t on Sunday. Let $f(8{:}00\text{AM}) = 0$ and $f(5{:}00\text{PM}) = M$. What are $g(8{:}00\text{AM})$ and $g(5{:}00\text{PM})$?

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