Math 1a. §5.1 Worksheet
Areas and Distances

Fall 2005

1. Write each of the following in summation notation.

   (a) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots 1000^2$

   (b) $1 - 3 + 5 - 7 + \cdots + 100$

   (c) $1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n}$

   (d) $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \cdots$, where

$$n! = \begin{cases} 1 \cdot 2 \cdot 3 \cdots n & n \geq 1, \\ 1 & n = 0. \end{cases}$$

2. Find an expression for the area under the graph of

$$f(x) = \frac{\ln x}{x},$$

   where $3 \leq x \leq 10$. You do not need to evaluate the limit.

3. Determine a region whose area is equal to

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(2 + \frac{5k}{n}\right)^3.$$

4. Oil pipelines are often hundreds of miles long. Their enormous length increases the tendency to leak, and finding these leaks can often be very difficult. The rate at which oil flows through a pipeline depends on the oil's viscosity, which in turn depends on the oils temperature. The higher the temperature, the lower the viscosity, and the oil will travel faster through the pipeline.

   If oil is pumped across a desert, temperatures may range from 2°C to 40°C over a 24 hour period. Decreases in volume at the refinery are often impossible to distinguish from decreases due to temperature variation. The address this problem, an oil company can make two types of measurements.

   (a) They measure the amount of oil pumped into the tanks at the refinery.

   (b) Meters to measure the rate at which oil is flowing can be placed at pumping stations along the pipeline.

   Over a 24 hour period an engineer at one station records the following measurements.

Time t (hours)	0	4	8	12	16	20	24
Rate of Flow $r(t)$ (barrels/hour)	32	31	37	50	52	42	33

   We wish to determine the possibility of a leak if the refinery receives 940 barrels of oil in the same 24 hour period. Assume that oil is always flowing in the pipeline. Most often engineers tolerate a error of about 2% before searching for leaks. Is there evidence of a leak in this scenario?

5. The electrical energy from an electrical power plant is measured in volt-amperes or kilowatt hours. The voltage on the main electrical carrier, which is rated at 2000 amperes, from a power plant in Rhode Island varies during the summer when there is a major drainon electricity due to air conditioning. The table below records the voltages in thousands of volts at one-hour intervals for four hours during a hot afternoon. Estimate the amount of energy that passed through this carrier.

Time	0	1	2	3	4
Volts (in thousands)	150	200	210	240	270