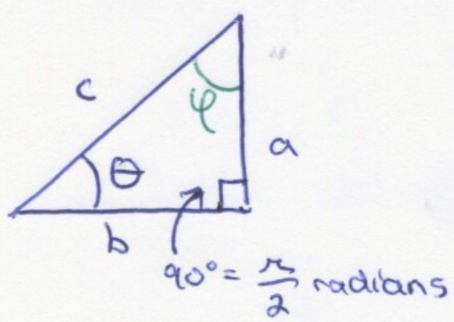
Math 1a: Section 10/3/05
Nicole's Notes

Topic: Calculating Trig Stuff Without Using a Calculator

Part I: If we're given a right triangle….



$a^2+b^2=c^2 \Rightarrow c=\sqrt{a^2+b^2}$

$\sin\theta = \frac{a}{c} \Rightarrow \theta = \sin^{-1}\left(\frac{a}{c}\right)$

$\cos\theta = \frac{b}{c} \Rightarrow \theta = \cos^{-1}\left(\frac{b}{c}\right)$

$\tan\theta = \frac{a}{b} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{b}\right)$

"arcsin" means "$\sin^{-1}$"

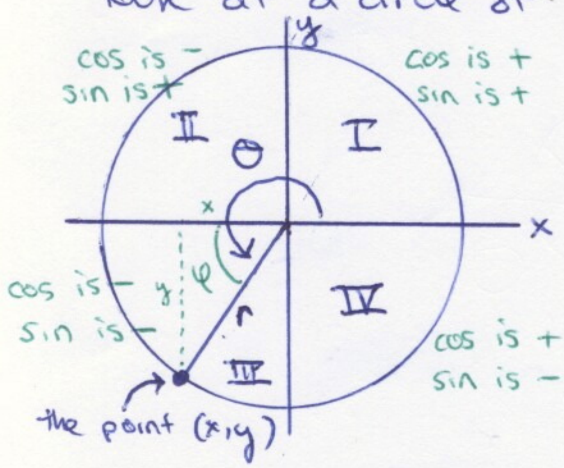
"arccos" means "$\cos^{-1}$"

"arctan" means "$\tan^{-1}$"

Can you find $\sin\varphi, \cos\varphi, \tan\varphi$?

Part II: If we're given a circle….

Last time we looked at the unit circle. This time, let's look at a circle of radius r.



$r=\sqrt{x^2+y^2}$ $\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$

1. Draw a reference angle, φ, and a triangle
2. The function values (sin, cos, tan) of the reference angle are the same as the function values of θ, give or take a plus or minus sign

   $\sin\theta = \pm\frac{y}{r}, \quad \cos\theta = \pm\frac{x}{r}, \quad \tan\theta = \pm\frac{y}{x}$

3. Remember whether it is + or − using the unit circle, for which $(x,y)=(\cos\theta, \sin\theta)$. So if x is negative, $\cos\theta$ is negative, etc.

4. If asked to give the angle θ, take $\cos^{-1}[\cos\theta]$ or $\sin^{-1}[\sin\theta]$. If you have a choice, take whichever answer that would put you in the correct quadrant. (see identities on pg. 2 to see why)

summary: add appropriate amount to reference angle

I: use $\cos^{-1}$ or $\sin^{-1}$ II: use $\cos^{-1}$ III: adjust quad II angle IV: adjust quad II/III angle

Part II: If we're given a circle, continued….

(4) (Continued) The <u>best</u> way to find θ is to find the reference angle, then add the appropriate amount to it. For example, in quad III, $\cos^{-1}\left(\frac{+x}{r}\right) = \varphi$, and $\theta = \pi + \varphi$

(5) If given a multiple choice question with different choices for θ, go through each one until you find one that makes sense.

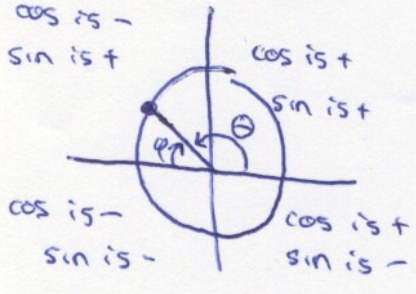
<u>identities</u>: $\sin(\pi-\theta) = \sin\theta$ $\sin(-\theta) = -\sin\theta$ $\cos(\pi+\theta) = -\cos\theta$

$\sin(\pi+\theta = -\sin\theta$ $\cos(\pi-\theta) = -\cos\theta$ $\cos(-\theta) = +\cos\theta$

Part III: If we aren't given a picture (oh no!)….

If you aren't given a picture… draw one!

If you are asked to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, draw θ on the unit circle, and use the reference angle to find the value of the function.

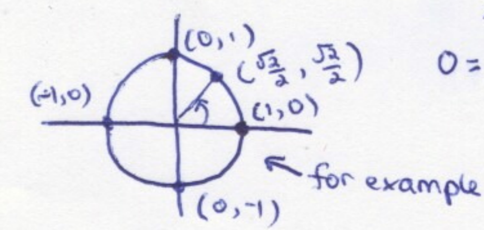


In this picture, $\sin\theta = \sin\varphi$
$\cos\theta = -\cos\varphi$

You should know some key values of $\sin\theta$ and $\cos\theta$:

radians	degrees	$\sin\theta$	$\cos\theta$
$\frac{\pi}{6}$	30	$\frac{1}{2}$	$\sqrt{3}/2$
$\pi/4$	45	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	60	$\sqrt{3}/2$	$1/2$
$\pi/2$	90	1	0
π	180	0	-1
$0 = 2\pi$	0	0	1
		(y-coor.)	(x-coor.)

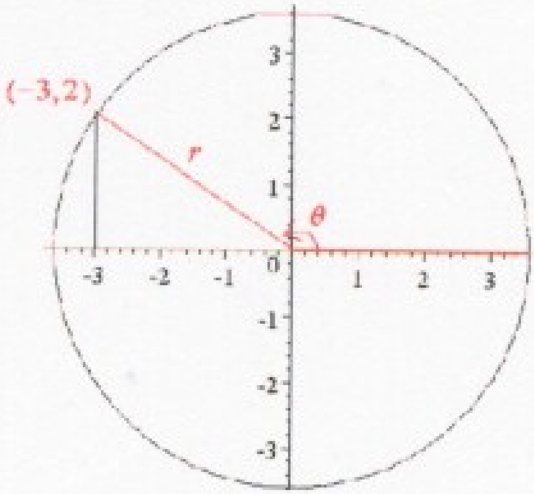
You can remember these values using the unit circle.



Part IV: Practice Problems

This is taken directly off the Practice PreCalculus Exam

18. The numerical value of θ is approximately equal to:



(a) $\arctan(\frac{-3}{2})$

(b) $\arctan(\frac{2}{-3})$

(c) $\arccos(\frac{-3}{\sqrt{13}})$
correct answer

(d) $\arcsin(\frac{2}{\sqrt{13}})$

(e) None of the above.

We arrive at this answer by going through each choice:

a) $\arctan\left(\frac{-3}{2}\right)$
b) $\arctan\left(\frac{2}{-3}\right)$
Remember that arctan looks like this:

So $\arctan$(negative number) = (negative angle)

But we <u>don't</u> want a negative angle!

✓ c) $\arccos\left(\frac{-3}{\sqrt{13}}\right)$ works, since $r = \sqrt{13}$, and $\arccos\left(\frac{-x}{\underbrace{\sqrt{x^2+y^2}}_{=r}}\right)$ will put us in the right quadrant

d) $\arcsin\left(\frac{2}{\sqrt{13}}\right)$ would give us an <u>acute</u> angle, since arcsin (positive number) gives the appropriate angle within a right triangle. But our angle is obtuse

This is also taken off the Practice Precalculus Exam

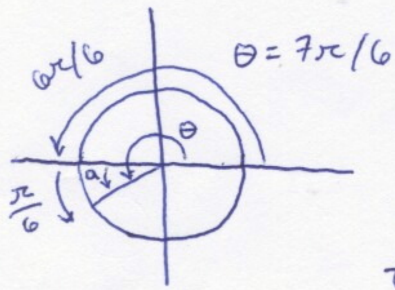
20. The value of $\tan(7\pi/6)$ is equal to:

(a) $\sqrt{3}/3$ correct answer
(b) $-\sqrt{3}/3$
(c) $\sqrt{3}$
(d) $-\sqrt{3}$
(e) None of the above.

We arrive at this answer like so:

$$7\pi/6 = 7 \times \frac{\pi}{6}, \text{ and } \frac{\pi}{6} = 30°$$

We draw the angle on the unit circle:



I have $\frac{7\pi}{6}$ drawn as $\frac{6\pi}{6} + \frac{\pi}{6}$
(note that $\frac{6\pi}{6} = \pi = 180°$)

. We can use the reference angle $a = \frac{\pi}{6}$
$\tan(a)$ will be the same as $\tan(\theta)$, give or take a plus or minus sign.

$$\tan(a) = \tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

need to know these by memory

$$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

So is $\tan(\theta)$ equal to $+\frac{\sqrt{3}}{3}$ or $-\frac{\sqrt{3}}{3}$?

In the third quadrant, $\sin\theta$ is negative, and $\cos\theta$ is ~~positive~~ negative (oops), so $\tan\theta = \frac{\sin\theta}{\cos\theta}$ is $\frac{\text{negative}}{\text{negative}}$, so $\tan\theta$ is positive. So $\tan\theta = +\frac{\sqrt{3}}{3}$