

Math 1a: Section 9/29/05 (Thursday 9-10:30 pm, SC 507)

Nicole's Notes

Pre-calculus Exam Overview

- Make sure to try the practice exam online
- The exam will be about 1/3 basic algebra/functions, 1/3 trigonometry, and 1/3 exponents/logs
- The exam will be multiple choice
- You will have up to 1.5 hours to complete the exam
- The exam will probably take you around 30 minutes
- NO calculators allowed
- You can use scratch paper if you want
- Go and take the pre-calculus exam during one of the CA sections next week in the computer labs – check online for the schedule
- You can't take the real exam at your dorm computer, because you need a CA to enter a proctor password

Pre-calculus Review

note: check out the front inside cover of your book for more formulas

Definition of a function

- A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B
- A is called the domain, B is called the range

The vertical line test

- A curve in the xy -plane is the graph of a function of x if and only if no vertical line intersects the curve more than once

The horizontal line test

- A function f is called one-to-one if it never takes on the same value twice
- A curve in the xy -plane is the graph of a one-to-one function if and only if no horizontal line intersects the curve more than once

Symmetry

- A function is EVEN if $f(-x) = f(x)$ for every number x in its domain
- A function is ODD if $f(-x) = -f(x)$ for every number x in its domain

Linear Functions

- We can write them in the form $y = f(x) = mx + b$
- m is the slope, b is the y -intercept

Polynomials

- We can write them in the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$
- n is a non-negative integer
- a_n, \dots, a_0 are constants called the coefficients
- n is the degree of the polynomial

Inverse Functions

- Let f be a one-to-one function with domain A and range B . The its inverse function f^{-1} has domain B and range A and is defined by:
 - $f^{-1}(y) = x$ if and only if $f(x) = y$
- To solve for the inverse in some equation $y = f(x)$, solve for x to get $f^{-1}(y)$:
 - $y = f(x) = 2x$
 - Solve for x to get $x = \frac{1}{2} y = f^{-1}(y)$
- To graphically find the inverse, reflect $f(x)$ across the line $y = x$

Exponential functions

- These come in the form $f(x) = a^x$

Properties of Exponents

- $a^{x+y} = a^x a^y$
- $a^{x-y} = (a^x)/(a^y)$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

Logarithms

- $\log_a x = y$ means that $a^y = x$
- $\log_a x$ and a^x are inverse functions:
 - $B^{\log_b A} = A$
 - $\log_A(A^x) = x$

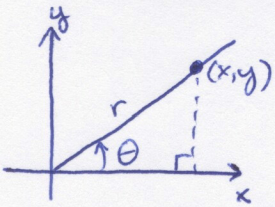
Properties of Logarithms

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(x/y) = \log_a x - \log_a y$
- $\log_a x^y = y \log_a x$
- $\log_a x = (\log_b x) / (\log_b a)$

e^x and the natural logarithm

- The number $e = 2.71828\dots$ shows up a lot in math problems involving the real world, so we gave it its own symbol, and its own logarithm
- $\log_e x = \ln(x)$

Trigonometric Functions

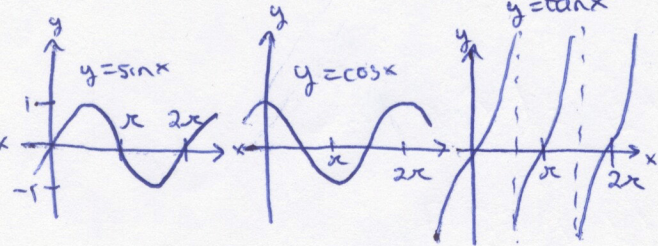


$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

Check out the graphs in the reference pages in your textbook, since book's graphs are better than mine



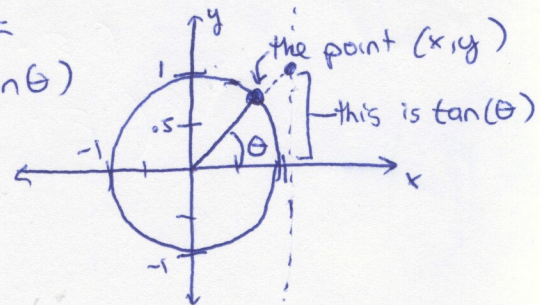
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\pi \text{ radians} = 180^\circ$$

The unit circle:

This is a way of finding $\sin \theta$, $\cos \theta$, and $\tan \theta$ all on one graph!

$$(x, y) = (\cos \theta, \sin \theta)$$



Transformations of Functions

- Let $c > 0$
 - $y = f(x) + c$ Shifts $f(x)$ c units up
 - $y = f(x) - c$ Shifts $f(x)$ c units down
 - $y = f(x - c)$ Shifts $f(x)$ c units right
 - $y = f(x + c)$ Shifts $f(x)$ c units left
- Let $c > 1$
 - $y = cf(x)$ Stretch vertically by factor c
 - $y = (1/c)f(x)$ Compress vertically by factor c
 - $y = f(cx)$ Compress horizontally by factor c
 - $y = f(x/c)$ Stretch horizontally by factor c
 - $y = -f(x)$ Reflect about the x -axis
 - $y = f(-x)$ Reflect about the y -axis

Quadratic formula

$$\text{If } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

as long as this is non-negative