Math 1a
Midterm I Review

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Resources for Review

* Midterm I Review Guide
  http://www.courses.fas.harvard.edu/~mathla/exams/midterm1f05review.pdf
* Exams and solutions from previous years http://www.courses.fas.harvard.edu/~mathla/prevexams/
* Solutions to the Chapter Review Exercises http://www.courses.fas.harvard.edu/~mathla/exams/

Exam Particulars

* Wednesday, October 26 at 5-7 PM in Science Center B
* No calculators allowed
* All out-of-sequence exams must be approved by the course head
* No make-up exams

What to Expect

* Approximately ten questions (some with several parts)
* The emphasis will be on material from Sections 2.1-2.9 in Chapter 2
* Refer to the Midterm I Review Guide for details

The Definition of the Derivative

Using the definition of the derivative, find $f'(x)$ if

$$f(x) = \frac{x^2+1}{x-2}.$$

The Meaning of the Derivative

The cost of producing x ounces of gold is from a new gold mine is $C(x)$ dollars.

(a) What is the meaning of the derivative $C'(x)$? What are its units?

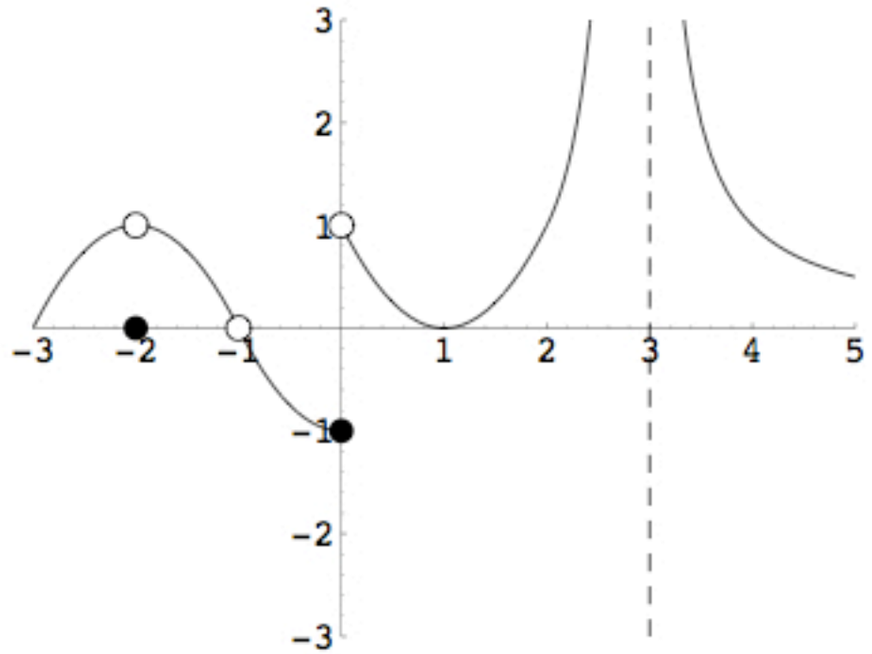
(b) What does the statement $C'(800) = 17$ mean?

(c) Do you think that the values of $C'(x)$ will increase or decrease in the sort term? What about in the long term? Explain.

(d) Which weighs more? A pound of gold or a pound of feathers?

Finding Limits

Given the graph of f, find each limit below or explain why the limit does not exist.



(a) $\lim_{x \to 2} f(x)$

(b) $\lim_{x \to -2} f(x)$

(c) $\lim_{x \to 3^+} f(x)$

(d) $\lim_{x \to 0} f(x)$

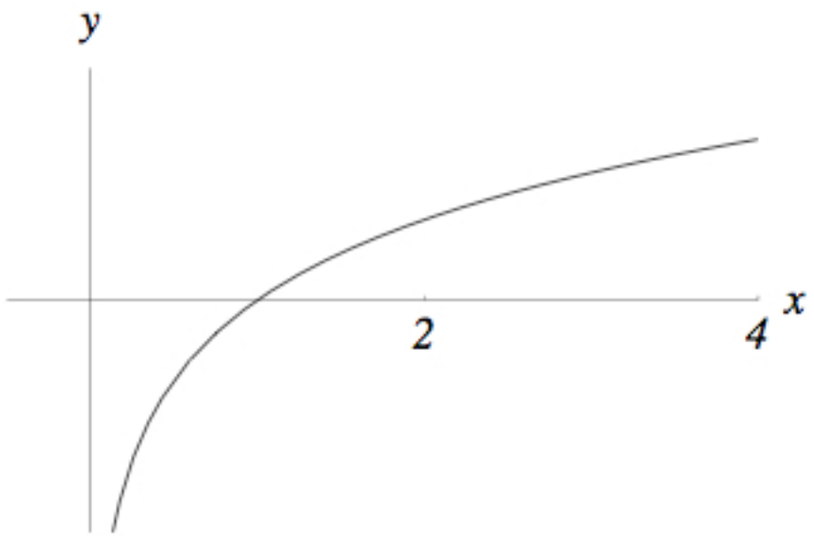
(e) $\lim_{x \to 0^+} f(x)$

(f) $\lim_{x \to 0^-} f(x)$

Secants and Tangents

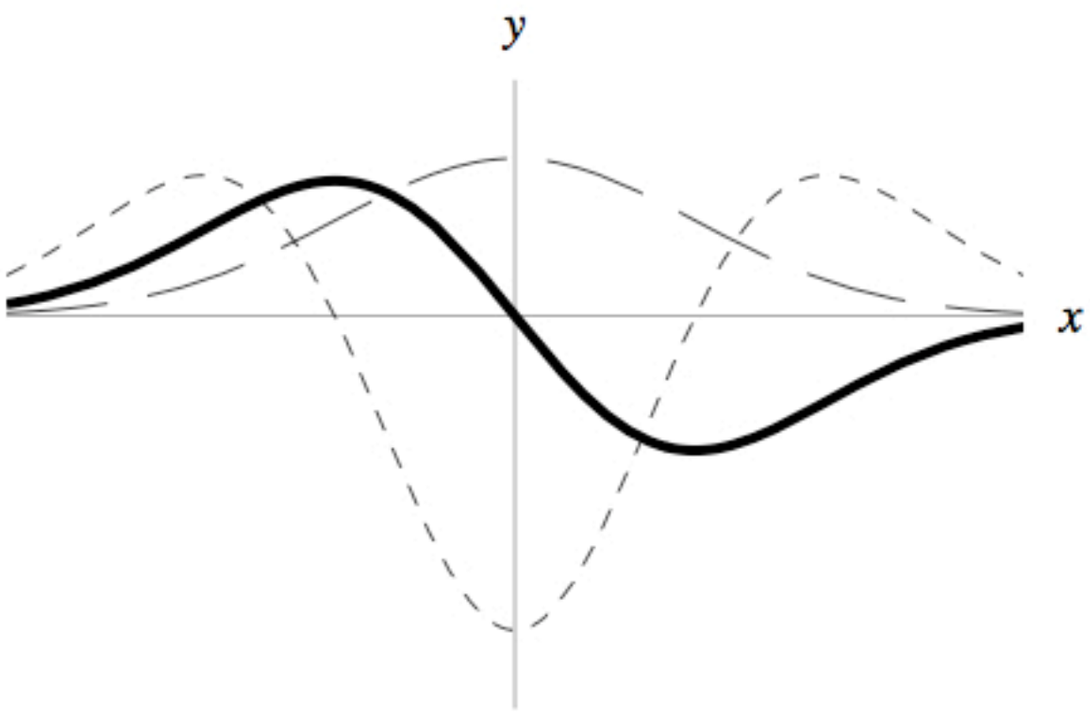
For the function whose graph is shown below, arrange the following numbers in increasing order and explain your reasoning.

$$0 \qquad f'(2) \qquad f(3) - f(2) \qquad \frac{1}{2}[f(4) - f(2)]$$



Finding f, f', and f''

The figure below shows the graphs of f, f', and f''. Identify each curve and explain your choices.



Sketching Graphs

Sketch the graph of a function that satisfies the given conditions:

- $f(0) = 0$
- $f'(-2) = f'(1) = f'(9) = 0$
- $\lim_{x \to \infty} = 0$ and $\lim_{x \to 6} = -\infty$
- $f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$
- $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$
- $f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$
- $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$

Continuity

For what value(s) of the constant c is the function

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

continuous on $(-\infty, \infty)$?