

# Math 1a

# Final Exam Review II

---

T. Judson  
January 9, 2006

# Resources for Review

- \* **Final Exam Review Guide**

<http://www.courses.fas.harvard.edu/~math1a/exams/finalf05review.pdf.pdf>

- \* **Exams and solutions from previous years**

<http://www.courses.fas.harvard.edu/~math1a/prevexams/>

- \* **Solutions to the Chapter Review Exercises**

<http://www.courses.fas.harvard.edu/~math1a/exams/>

# Exam Particulars

- \* Saturday, January 14 at 9:15 AM to 12:15 PM
- \* Students whose last name begins with A-L will take the exam in Science Center C
- \* Students whose name begins with M-Z will take the exam in Science Center B.
- \* No calculators allowed
- \* All out-of-sequence exams must be approved by the Final Exams Office

# What to Expect

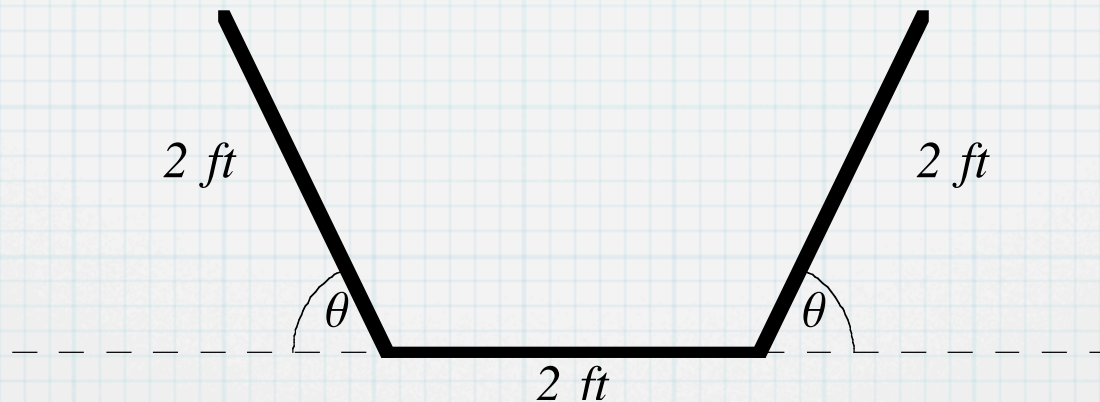
- \* Approximately fifteen questions (some with several parts)
- \* The Exam is comprehensive
- \* Refer to the Final Exam Review Guide for details

# A Strategy for Optimization

- \* Draw the picture.
- \* Name the variables.
- \* Write down the function that needs to be maximized or minimized.
- \* Write down the relationships between the variables.
- \* Reduce the function that is to be optimized to a function in one variable using the relationships in the previous step.
- \* Find the critical values of the function.
- \* Apply the First or Second Derivative Test to the Function. Don't forget to test the endpoints.
- \* Write down the final answer.

# An Optimization Problem

A trough that is 20 feet long is to be constructed from a sheet of metal of width 6 feet by bending up one-third of the sheet on each side through an angle  $\theta$ . How should  $\theta$  be chosen so that the trough will hold the maximum amount of water? (10 points)



# A Strategy for Related Rates

- \* Read the problem and draw a picture.
- \* Name the variables. These are the quantities that are changing.
- \* Write down the rates of change that you know and the rates of change that you want to know.
- \* Write down an equation that relates the variables.
- \* Implicitly differentiate the equation with respect to the appropriate variable (usually time).
- \* Substitute the quantities you know into the equation from the previous step.
- \* Solve for any unknown variables.
- \* Write down the final answer. Does your answer make sense?

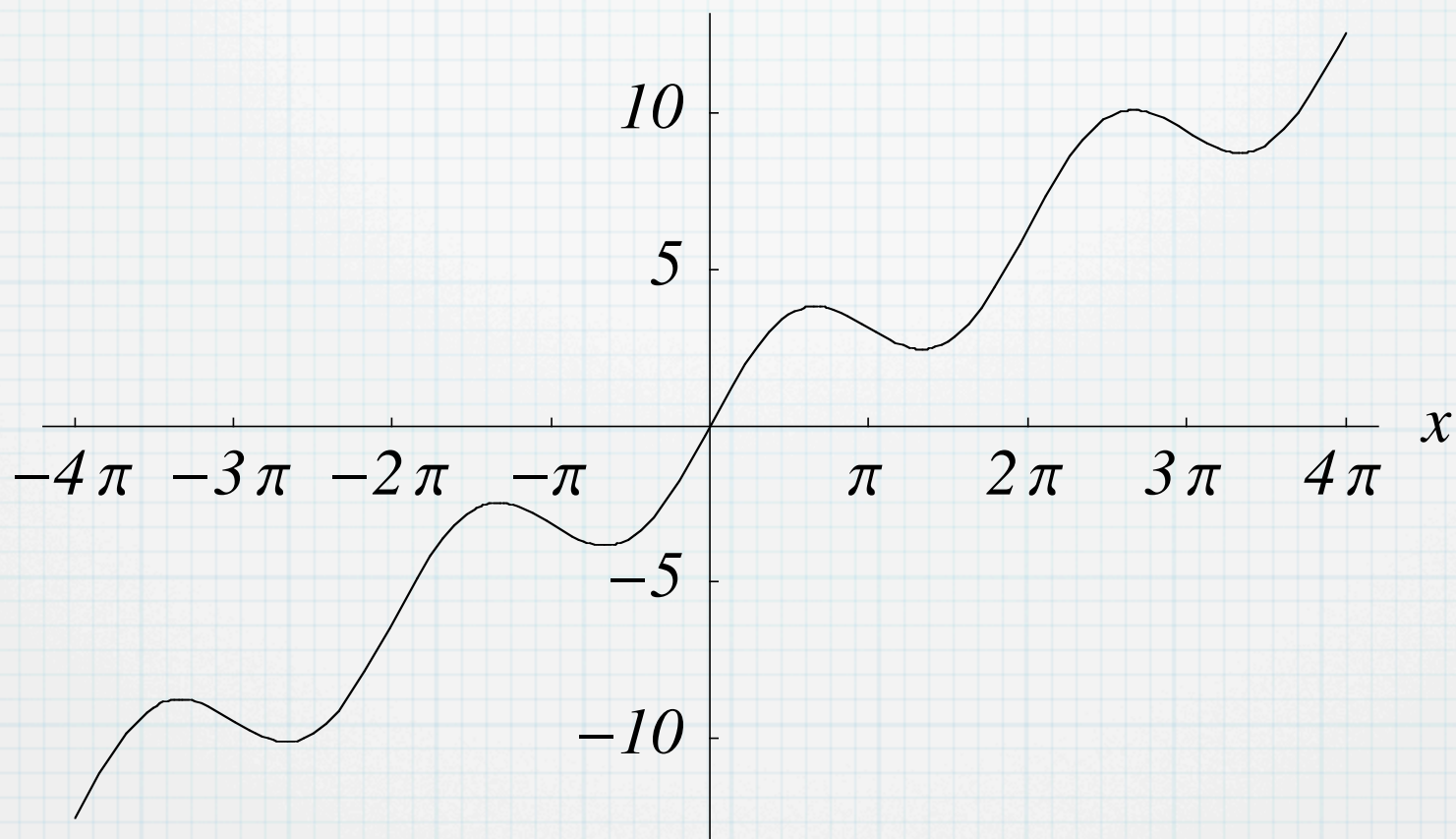
# Related Rates

Two ships sail from the same island port, one going north at 16 knots and the other east at 20 knots. The northbound ship departed at 9:00 A.M. and the eastbound ship left at 11:00 A.M. How fast is the distance between them increasing at 2:00 P.M.?

# Curve Sketching

$$f(x) = x + 2 \sin x$$

$$x + 2\sin x$$



# L'Hospital's Rule

If

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

is of the form  $0/0$  or  $\infty/\infty$  and

$$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

exists, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

L'Hôpital's rule holds if  $a$  is replaced by  $\infty$ ,  $-\infty$ ,  $a^+$ , or  $a^-$ .

# Some Examples

- $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{4x + 3x^2}$

- $\lim_{x \rightarrow 0^+} x^2 \ln x$

- $\lim_{x \rightarrow 1} (e^x + x)^{1/x}$