Name: ___________________ Class:___ Date: _____________

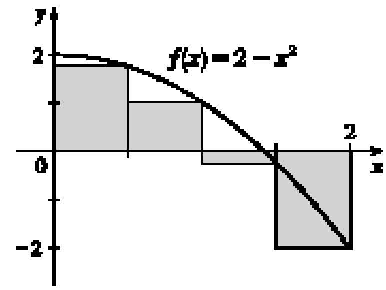
1. $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \quad [x_i^* = x_i \text{ is a right endpoint and } \Delta x = 0.5]$

$= 0.5\,[f(0.5) + f(1) + f(1.5) + f(2)] \quad [f(x) = 2 - x^2]$

$= 0.5\,[1.75 + 1 + (-0.25) + (-2)]$

$= 0.5(0.5) = 0.25$

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.



6. (a) Using the right endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_i)\,\Delta x = 1[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)]$$
$$\approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5$$

(b) Using the left endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_{i-1})\,\Delta x = 1[g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)]$$
$$\approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5 = -1$$

(c) Using the midpoint of each subinterval to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(\overline{x}_i)\,\Delta x = 1[g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5)]$$
$$\approx 1.5 + 0 - 1 - 1.75 - 1 + 0.5 = -1.75$$

9. $\Delta x = (10 - 2)/4 = 2$, so the endpoints are 2, 4, 6, 8, and 10, and the midpoints are 3, 5, 7, and 9. The Midpoint Rule gives $\int_2^{10} \sqrt{x^3 + 1}\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 2\left(\sqrt{3^3 + 1} + \sqrt{5^3 + 1} + \sqrt{7^3 + 1} + \sqrt{9^3 + 1}\right) \approx 124.1644.$

20. On $[0, 2]$, $\lim_{n\to\infty} \sum_{i=1}^{n} [4 - 3(x_i^*)^2 + 6(x_i^*)^5]\,\Delta x = \int_0^2 (4 - 3x^2 + 6x^5)\,dx.$

23. Note that $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$ and $x_i = 0 + i\,\Delta x = \dfrac{2i}{n}$.

$$\int_0^2 (2-x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left(2-\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n} 2 - \frac{4}{n^2}\sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty}\frac{2}{n}\left(2n - \frac{4}{n^2}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\left[4 - \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty}\left(4 - \frac{4}{3}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty}\left[4 - \frac{4}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 4 - \tfrac{4}{3}\cdot 1\cdot 2 = \tfrac{4}{3}$$

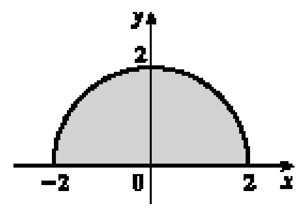
32. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ (area of a triangle)

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ (negative of the area of a semicircle)

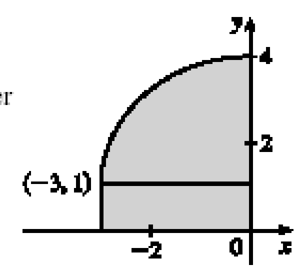
(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ (area of a triangle)

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

34. $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{4-x^2}$ between $x=-2$ and $x=2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{1}{2}\pi\cdot 2^2 = 2\pi$.



35. $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx$ can be interpreted as the area under the graph of $f(x) = 1+\sqrt{9-x^2}$ between $x=-3$ and $x=0$. This is equal to one-quarter the area of the circle with radius 3, plus the area of the rectangle, so $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx = \frac{1}{4}\pi\cdot 3^2 + 1\cdot 3 = 3 + \frac{9}{4}\pi$.



41. $\int_{-2}^{2} f(x)\,dx + \int_2^5 f(x)\,dx - \int_{-2}^{-1} f(x)\,dx = \int_{-2}^{5} f(x)\,dx + \int_{-1}^{-2} f(x)\,dx$ [by Property 5 and reversing limits]

$= \int_{-1}^{5} f(x)\,dx$ [Property 5]

42. $\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$

43. $\int_0^9 [2f(x) + 3g(x)]\,dx = 2\int_0^9 f(x)\,dx + 3\int_0^9 g(x)\,dx = 2(37) + 3(16) = 122$

50. If $0 \le x \le 2$, then $0 \le x^3 \le 8$, so $1 \le x^3 + 1 \le 9$ and $1 \le \sqrt{x^3+1} \le 3$.
Thus, $1(2-0) \le \int_0^2 \sqrt{x^3+1}\,dx \le 3(2-0)$; that is, $2 \le \int_0^2 \sqrt{x^3+1}\,dx \le 6$.

51. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{i^4}{n^5} = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{i^4}{n^4} \cdot \frac{1}{n} = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\frac{i}{n}\right)^4 \frac{1}{n}$. At this point, we need to recognize the limit as being of the form $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(x_i)\,\Delta x$, where $\Delta x = (1-0)/n = 1/n$, $x_i = 0 + i\,\Delta x = i/n$, and $f(x) = x^4$. Thus, the definite integral is $\int_0^1 x^4\,dx$.