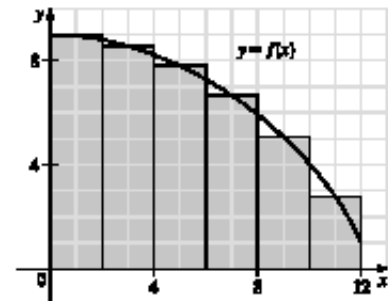
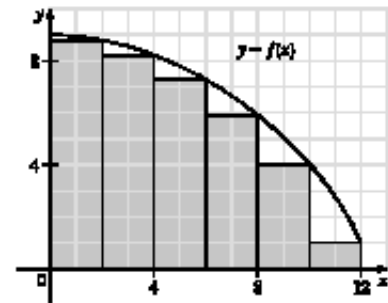
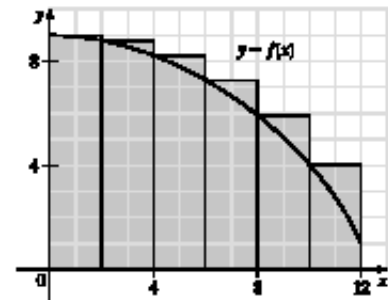


$$\begin{aligned}
 2. (a) (i) L_6 &= \sum_{i=1}^6 f(x_{i-1})\Delta x \quad [\Delta x = \frac{12-0}{6} = 2] \\
 &= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\
 &= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)] \\
 &\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) \\
 &= 2(43.3) = 86.6
 \end{aligned}$$

$$\begin{aligned}
 (ii) R_6 &= L_6 + 2 \cdot f(12) - 2 \cdot f(0) \\
 &\approx 86.6 + 2(1) - 2(9) = 70.6
 \end{aligned}$$

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]

$$\begin{aligned}
 (iii) M_6 &= \sum_{i=1}^6 f(x_i^*) \Delta x \\
 &= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \\
 &\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \\
 &= 2(39.7) = 79.4
 \end{aligned}$$



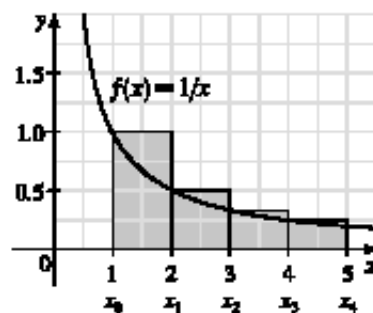
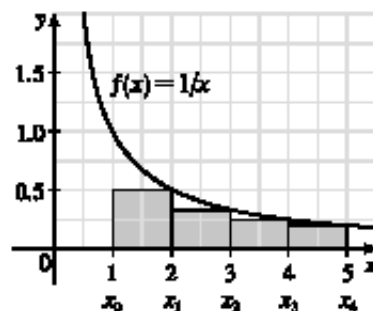
- (b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6 .
- (c) Since f is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is, R_6 .
- (d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6 .

$$\begin{aligned}
 3. \text{ (a) } R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \quad [\Delta x = \frac{5-1}{4} = 1] \\
 &= f(x_1) \cdot 1 + f(x_2) \cdot 1 + f(x_3) \cdot 1 + f(x_4) \cdot 1 \\
 &= f(2) + f(3) + f(4) + f(5) \\
 &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} = 1.28\bar{3}
 \end{aligned}$$

Since f is *decreasing* on $[1, 5]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4 .

$$\begin{aligned}
 \text{(b) } L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x \\
 &= f(1) + f(2) + f(3) + f(4) \\
 &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} = 2.08\bar{3}
 \end{aligned}$$

L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(1) \cdot 1 - f(5) \cdot 1$.



13. Lower estimate for oil leakage: $R_5 = (7.6 + 6.8 + 6.2 + 5.7 + 5.3)(2) = (31.6)(2) = 63.2$ L.

Upper estimate for oil leakage: $L_5 = (8.7 + 7.6 + 6.8 + 6.2 + 5.7)(2) = (35)(2) = 70$ L.

14. We can find an upper estimate by using the final velocity for each time interval. Thus, the distance d traveled after 62 seconds can be approximated by

$$d = \sum_{i=1}^6 v(t_i) \Delta t_i = (185 \text{ ft/s})(10 \text{ s}) + 319 \cdot 5 + 447 \cdot 5 + 742 \cdot 12 + 1325 \cdot 27 + 1445 \cdot 3 = 54,694 \text{ ft}$$

15. For a decreasing function, using left endpoints gives us an overestimate and using right endpoints results in an underestimate. We will use M_6 to get an estimate. $\Delta t = 1$, so

$$\begin{aligned}
 M_6 &= 1[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5) + v(5.5)] \\
 &\approx 55 + 40 + 28 + 18 + 10 + 4 = 155 \text{ ft}
 \end{aligned}$$

For a very rough check on the above calculation, we can draw a line from $(0, 70)$ to $(6, 0)$ and calculate the area of the triangle: $\frac{1}{2}(70)(6) = 210$. This is clearly an overestimate, so our midpoint estimate of 155 is reasonable.

17. $f(x) = \sqrt[4]{x}$, $1 \leq x \leq 16$. $\Delta x = (16 - 1)/n = 15/n$ and $x_i = 1 + i \Delta x = 1 + 15i/n$.

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt[4]{1 + \frac{15i}{n}} \cdot \frac{15}{n}.$$

19. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \tan \frac{i\pi}{4n}$ can be interpreted as the area of the region lying under the graph of $y = \tan x$ on the interval $[0, \frac{\pi}{4}]$,

since for $y = \tan x$ on $[0, \frac{\pi}{4}]$ with $\Delta x = \frac{\pi/4 - 0}{n} = \frac{\pi}{4n}$, $x_i = 0 + i \Delta x = \frac{i\pi}{4n}$, and $x_i^* = x_i$, the expression for the area is

$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \tan\left(\frac{i\pi}{4n}\right) \frac{\pi}{4n}$. Note that this answer is not unique, since the expression for the area is

the same for the function $y = \tan(x - k\pi)$ on the interval $[k\pi, k\pi + \frac{\pi}{4}]$, where k is any integer.

20. (a) $\Delta x = \frac{1-0}{n} = \frac{1}{n}$ and $x_i = 0 + i \Delta x = \frac{i}{n}$. $A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n}$.

(b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^3} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1}{n^4} \sum_{i=1}^n i^3 = \lim_{n \rightarrow \infty} \frac{1}{n^4} \left[\frac{n(n+1)}{2}\right]^2 = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{4n^2} = \frac{1}{4} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^2 = \frac{1}{4}$