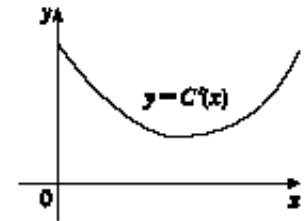


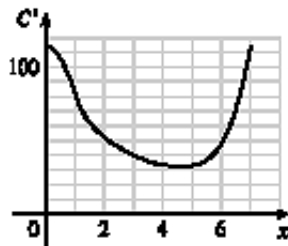
1. (a) $C(0)$ represents the fixed costs of production, such as rent, utilities, machinery etc., which are incurred even when nothing is produced.

(b) The inflection point is the point at which $C''(x)$ changes from negative to positive; that is, the marginal cost $C'(x)$ changes from decreasing to increasing. Thus, the marginal cost is minimized at the inflection point.

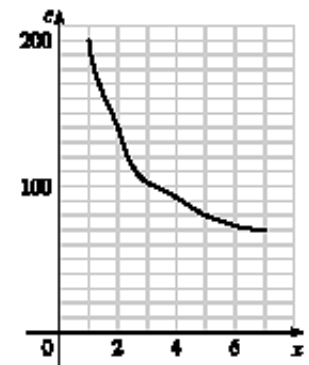
(c) The marginal cost function is $C'(x)$. We graph it as in Example 1 in Section 2.8.



2. (a) We graph C' as in Example 1 in Section 2.8.



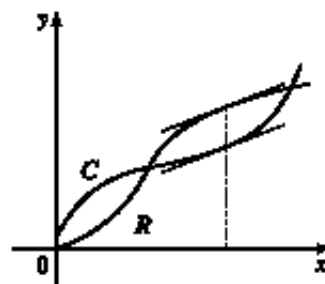
(b) By reading values of $C(x)$ from its graph, we can plot $c(x) = C(x)/x$.



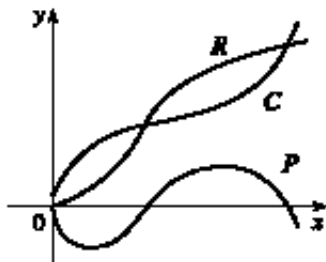
(c) Since the graph in part (b) is decreasing, we estimate that the minimum value of $c(x)$ occurs at $x = 7$. The average cost and the marginal cost are equal at that value. See the box preceding Example 1.

3. $c(x) = 21.4 - 0.002x$ and $c(x) = C(x)/x \Rightarrow C(x) = 21.4x - 0.002x^2$. $C'(x) = 21.4 - 0.004x$ and $C'(1000) = 17.4$. This means that the cost of producing the 1001st unit is about \$17.40.

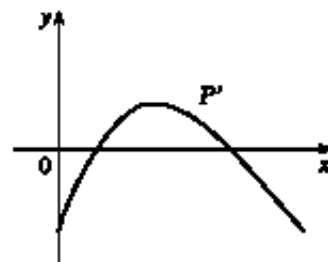
4. (a) Profit is maximized when the marginal revenue is equal to the marginal cost; that is, when R and C have equal slopes. See the box preceding Example 2.



- (b) $P(x) = R(x) - C(x)$ is sketched.



- (c) The marginal profit function is defined as $P'(x)$.



5. (a) $C(x) = 16,000 + 200x + 4x^{3/2}$, $C(1000) = 16,000 + 200,000 + 40,000\sqrt{10} \approx 216,000 + 126,491$, so $C(1000) \approx \$342,491$. $c(x) = C(x)/x = \frac{16,000}{x} + 200 + 4x^{1/2}$, $c(1000) \approx \$342.49/\text{unit}$. $C'(x) = 200 + 6x^{1/2}$, $C'(1000) = 200 + 60\sqrt{10} \approx \$389.74/\text{unit}$.

- (b) We must have $C'(x) = c(x) \Leftrightarrow 200 + 6x^{1/2} = \frac{16,000}{x} + 200 + 4x^{1/2} \Leftrightarrow 2x^{3/2} = 16,000 \Leftrightarrow$

$x = (8,000)^{2/3} = 400$ units. To check that this is a minimum, we calculate

$$c'(x) = \frac{-16,000}{x^2} + \frac{2}{\sqrt{x}} = \frac{2}{x^2}(x^{3/2} - 8000).$$

This is negative for $x < (8000)^{2/3} = 400$, zero at $x = 400$, and positive for $x > 400$, so c is decreasing on $(0, 400)$ and increasing on $(400, \infty)$. Thus, c has an absolute minimum at $x = 400$.

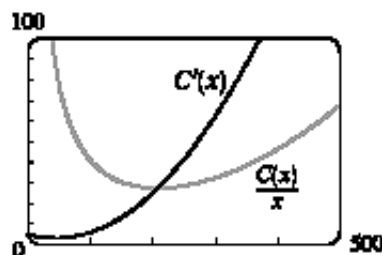
[Note: $c''(x)$ is *not* positive for all $x > 0$.]

- (c) The minimum average cost is $c(400) = 40 + 200 + 80 = \$320/\text{unit}$.

7. (a) $C(x) = 3700 + 5x - 0.04x^2 + 0.0003x^3 \Rightarrow C'(x) = 5 - 0.08x + 0.0009x^2$ (marginal cost).

$$c(x) = \frac{C(x)}{x} = \frac{3700}{x} + 5 - 0.04x + 0.0003x^2 \text{ (average cost).}$$

(b)



The graphs intersect at (208.51, 27.45), so the production level that minimizes average cost is about 209 units.

(c) $c'(x) = -\frac{3700}{x^2} - 0.04 + 0.0006x = 0 \Rightarrow 3700 + 0.04x^2 - 0.0006x^3 = 0 \Rightarrow x_1 \approx 208.51$.

$$c(x_1) \approx \$27.45/\text{unit.}$$

(d) The marginal cost is given by $C'(x)$, so to find its minimum value we'll find the derivative of C' ; that is, C'' .

$$C''(x) = -0.08 + 0.0018x = 0 \Rightarrow x_1 = \frac{800}{18} = 44.4\bar{4}. \quad C'(x_1) = \$3.22/\text{unit.}$$

$C'''(x) = 0.0018 > 0$ for all x , so this is the minimum marginal cost. C''' is the second derivative of C' .

10. $C(x) = 16,000 + 500x - 1.6x^2 + 0.004x^3$, $p(x) = 1700 - 7x$. Then $R(x) = xp(x) = 1700x - 7x^2$. If the profit is maximum, then $R'(x) = C'(x) \Leftrightarrow 1700 - 14x = 500 - 3.2x + 0.012x^2 \Leftrightarrow 0.012x^2 + 10.8x - 1200 = 0 \Leftrightarrow x^2 + 900x - 100,000 = 0 \Leftrightarrow (x + 1000)(x - 100) = 0 \Leftrightarrow x = 100$ (since $x > 0$). The profit is maximized if $P''(x) < 0$, but since $P''(x) = R''(x) - C''(x)$, we can just check the condition $R''(x) < C''(x)$. Now $R''(x) = -14 < -3.2 + 0.024x = C''(x)$ for $x > 0$, so there is a maximum at $x = 100$.

15. (a) We are given that the demand function p is linear and $p(27,000) = 10$, $p(33,000) = 8$, so the slope is

$$\frac{10-8}{27,000-33,000} = -\frac{1}{3000} \text{ and an equation of the line is } y - 10 = \left(-\frac{1}{3000}\right)(x - 27,000) \Rightarrow$$

$$y = p(x) = -\frac{1}{3000}x + 19 = 19 - (x/3000).$$

(b) The revenue is $R(x) = xp(x) = 19x - (x^2/3000) \Rightarrow R'(x) = 19 - (x/1500) = 0$ when $x = 28,500$. Since $R''(x) = -1/1500 < 0$, the maximum revenue occurs when $x = 28,500 \Rightarrow$ the price is $p(28,500) = \$9.50$.

16. (a) Let $p(x)$ be the demand function. Then $p(x)$ is linear and $y = p(x)$ passes through (20, 10) and (18, 11), so the slope is $-\frac{1}{2}$ and an equation of the line is $y - 10 = -\frac{1}{2}(x - 20) \Leftrightarrow y = -\frac{1}{2}x + 20$. Thus, the demand is $p(x) = -\frac{1}{2}x + 20$ and the revenue is $R(x) = xp(x) = -\frac{1}{2}x^2 + 20x$.

(b) The cost is $C(x) = 6x$, so the profit is $P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 14x$. Then $0 = P'(x) = -x + 14 \Rightarrow x = 14$. Since $P''(x) = -1 < 0$, the selling price for maximum profit is $p(14) = -\frac{1}{2}(14) + 20 = \13 .

18. Let x denote the number of \$10 increases in rent. Then the price is $p(x) = 800 + 10x$, and the number of units occupied is $100 - x$. Now the revenue is

$$\begin{aligned} R(x) &= (\text{rental price per unit}) \times (\text{number of units rented}) \\ &= (800 + 10x)(100 - x) = -10x^2 + 200x + 80,000 \text{ for } 0 \leq x \leq 100 \Rightarrow \end{aligned}$$

$R'(x) = -20x + 200 = 0 \Leftrightarrow x = 10$. This is a maximum since $R''(x) = -20 < 0$ for all x . Now we must check the value of $R(x) = (800 + 10x)(100 - x)$ at $x = 10$ and at the endpoints of the domain to see which value of x gives the maximum value of R . $R(0) = 80,000$, $R(10) = (900)(90) = 81,000$, and $R(100) = (1800)(0) = 0$. Thus, the maximum revenue of \$81,000/week occurs when 90 units are occupied at a rent of \$900/week.

20. She will have A/n dollars after each withdrawal and 0 dollars just before the next withdrawal, so her average cash balance at any given time is $\frac{1}{2}(A/n + 0) = A/(2n)$. The transaction costs for n withdrawals are nT . The lost interest cost on the average cash balance is $[A/(2n)] \cdot R$. Thus, the total cost for n transactions is $C(n) = nT + \frac{AR}{2n}$. Now $C'(n) = T - \frac{AR}{2n^2}$

and $C'(n) = 0 \Rightarrow \frac{AR}{2n^2} = T \Rightarrow n^2 = \frac{AR}{2T} \Rightarrow n = \sqrt{\frac{AR}{2T}}$, the value of n that minimizes total costs since

$C''(n) = -\frac{AR}{n^3} < 0$. Thus, the optimal average cash balance is $\frac{A}{2n} = \frac{A\sqrt{2T}}{2\sqrt{AR}} = \frac{\sqrt{AT}}{\sqrt{2R}} = \sqrt{\frac{AT}{2R}}$.