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1. (a) $\lim_{x\to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{0}{0}$.

(b) $\lim_{x\to a} \frac{f(x)}{p(x)} = 0$ because the numerator approaches 0 while the denominator becomes large.

(c) $\lim_{x\to a} \frac{h(x)}{p(x)} = 0$ because the numerator approaches a finite number while the denominator becomes large.

(d) If $\lim_{x\to a} p(x) = \infty$ and $f(x) \to 0$ through positive values, then $\lim_{x\to a} \frac{p(x)}{f(x)} = \infty$. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = x^2$.] If $f(x) \to 0$ through negative values, then $\lim_{x\to a} \frac{p(x)}{f(x)} = -\infty$. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = -x^2$.] If $f(x) \to 0$ through both positive and negative values, then the limit might not exist. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = x$.]

(e) $\lim_{x\to a} \frac{p(x)}{q(x)}$ is an indeterminate form of type $\frac{\infty}{\infty}$.

4. (a) $\lim_{x\to a} [f(x)]^{g(x)}$ is an indeterminate form of type 0^0.

(b) If $y = [f(x)]^{p(x)}$, then $\ln y = p(x) \ln f(x)$. When x is near a, $p(x) \to \infty$ and $\ln f(x) \to -\infty$, so $\ln y \to -\infty$. Therefore, $\lim_{x\to a} [f(x)]^{p(x)} = \lim_{x\to a} y = \lim_{x\to a} e^{\ln y} = 0$, provided f^p is defined.

(c) $\lim_{x\to a} [h(x)]^{p(x)}$ is an indeterminate form of type 1^∞.

(d) $\lim_{x\to a} [p(x)]^{f(x)}$ is an indeterminate form of type ∞^0.

(e) If $y = [p(x)]^{q(x)}$, then $\ln y = q(x) \ln p(x)$. When x is near a, $q(x) \to \infty$ and $\ln p(x) \to \infty$, so $\ln y \to \infty$. Therefore, $\lim_{x\to a} [p(x)]^{q(x)} = \lim_{x\to a} y = \lim_{x\to a} e^{\ln y} = \infty$.

(f) $\lim_{x\to a} \sqrt[q(x)]{p(x)} = \lim_{x\to a} [p(x)]^{1/q(x)}$ is an indeterminate form of type ∞^0.

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{\text{H}}{=}$

7. This limit has the form $\frac{0}{0}$. $\lim_{x\to(\pi/2)^+} \frac{\cos x}{1-\sin x} \overset{\text{H}}{=} \lim_{x\to(\pi/2)^+} \frac{-\sin x}{-\cos x} = \lim_{x\to(\pi/2)^+} \tan x = -\infty$.

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{\text{H}}{=}$

9. This limit has the form $\frac{0}{0}$. $\lim_{t\to 0} \frac{e^t - 1}{t^3} \overset{\text{H}}{=} \lim_{t\to 0} \frac{e^t}{3t^2} = \infty$ since $e^t \to 1$ and $3t^2 \to 0^+$ as $t \to 0$.

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Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

14. $\lim\limits_{x\to\infty} \frac{\ln \ln x}{x} \overset{H}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \lim\limits_{x\to\infty} \frac{1}{x\ln x} = 0$

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

20. $\lim\limits_{x\to 0} \frac{\cos mx - \cos nx}{x^2} \overset{H}{=} \lim\limits_{x\to 0} \frac{-m\sin mx + n\sin nx}{2x} \overset{H}{=} \lim\limits_{x\to 0} \frac{-m^2\cos mx + n^2\cos nx}{2} = \frac{1}{2}(n^2 - m^2)$

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

26. $\lim\limits_{x\to -\infty} x^2 e^x = \lim\limits_{x\to -\infty} \frac{x^2}{e^{-x}} \overset{H}{=} \lim\limits_{x\to -\infty} \frac{2x}{-e^{-x}} \overset{H}{=} \lim\limits_{x\to -\infty} \frac{2}{e^{-x}} = \lim\limits_{x\to -\infty} 2e^x = 0$

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

33. The limit has the form $\infty - \infty$ and we will change the form to a product by factoring out x.

$\lim\limits_{x\to\infty}(x - \ln x) = \lim\limits_{x\to\infty} x\left(1 - \frac{\ln x}{x}\right) = \infty$ since $\lim\limits_{x\to\infty} \frac{\ln x}{x} \overset{H}{=} \lim\limits_{x\to\infty} \frac{1/x}{1} = 0.$

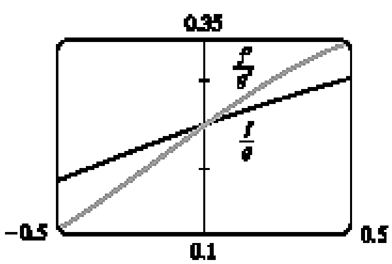
Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

37. $y = (1-2x)^{1/x} \;\Rightarrow\; \ln y = \frac{1}{x}\ln(1-2x)$, so $\lim\limits_{x\to 0}\ln y = \lim\limits_{x\to 0} \frac{\ln(1-2x)}{x} \overset{H}{=} \lim\limits_{x\to 0} \frac{-2/(1-2x)}{1} = -2 \;\Rightarrow$

$\lim\limits_{x\to 0}(1-2x)^{1/x} = \lim\limits_{x\to 0} e^{\ln y} = e^{-2}.$

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{H}{=}$

43.



From the graph, it appears that $\lim\limits_{x\to 0} \frac{f(x)}{g(x)} = \lim\limits_{x\to 0} \frac{f'(x)}{g'(x)} = 0.25$. We calculate $\lim\limits_{x\to 0} \frac{f(x)}{g(x)} = \lim\limits_{x\to 0} \frac{e^x - 1}{x^3 + 4x} \overset{H}{=} \lim\limits_{x\to 0} \frac{e^x}{3x^2 + 4} = \frac{1}{4}.$

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{\text{H}}{=}$

58. (a) $\lim_{t\to\infty} v = \lim_{t\to\infty} \frac{mg}{c}\left(1 - e^{-ct/m}\right) = \frac{mg}{c}\lim_{t\to\infty}\left(1 - e^{-ct/m}\right) = \frac{mg}{c}(1-0)$ [because $-ct/m \to -\infty$ as $t \to \infty$] $= \frac{mg}{c}$, which is the speed the object approaches as time goes on, the so-called limiting velocity.

(b) $\lim_{c\to 0^+} v = \lim_{c\to 0^+} \frac{mg}{c}\left(1 - e^{-ct/m}\right) = mg \lim_{c\to 0^+} \frac{1 - e^{-ct/m}}{c}$ [form is $\frac{0}{0}$]

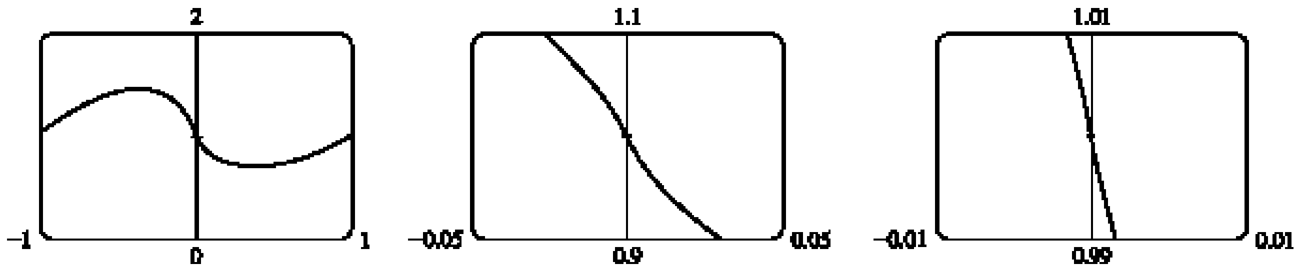
$$\overset{\text{H}}{=} mg \lim_{c\to 0^+} \frac{\left(-e^{-ct/m}\right)\cdot(-t/m)}{1} = \frac{mgt}{m}\lim_{c\to 0^+} e^{-ct/m} = gt(1) = gt$$

The velocity of a falling object in a vacuum is directly proportional to the amount of time it falls.

Note: The use of l'Hospital's Rule is indicated by an H above the equal sign: $\overset{\text{H}}{=}$

66. (a) For f to be continuous, we need $\lim_{x\to 0} f(x) = f(0) = 1$. We note that for $x \neq 0$, $\ln f(x) = \ln|x|^x = x\ln|x|$. So $\lim_{x\to 0}\ln f(x) = \lim_{x\to 0} x\ln|x| = \lim_{x\to 0}\frac{\ln|x|}{1/x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{1/x}{-1/x^2} = 0$. Therefore, $\lim_{x\to 0} f(x) = \lim_{x\to 0} e^{\ln f(x)} = e^0 = 1$. So f is continuous at 0.

(b) From the graphs, it appears that f is differentiable at 0.



(c) To find f', we use logarithmic differentiation: $\ln f(x) = x\ln|x| \Rightarrow \frac{f'(x)}{f(x)} = x\left(\frac{1}{x}\right) + \ln|x| \Rightarrow$ $f'(x) = f(x)(1 + \ln|x|) = |x|^x(1 + \ln|x|)$, $x \neq 0$. Now $f'(x) \to -\infty$ as $x \to 0$ [since $|x|^x \to 1$ and $(1 + \ln|x|) \to -\infty$], so the curve has a vertical tangent at $(0, 1)$ and is therefore not differentiable there. The fact cannot be seen in the graphs in part (b) because $\ln|x| \to -\infty$ very slowly as $x \to 0$.