

$$1. V = x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt} = 3x^2 \frac{dx}{dt}$$

$$2. (a) A = \pi r^2 \Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$$

$$4. A = \ell w \Rightarrow \frac{dA}{dt} = \ell \cdot \frac{dw}{dt} + w \cdot \frac{d\ell}{dt} = 20(3) + 10(8) = 140 \text{ cm}^2/\text{s}.$$

$$7. z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right). \text{ When } x = 5 \text{ and } y = 12,$$

$$z^2 = 5^2 + 12^2 \Rightarrow z^2 = 169 \Rightarrow z = \pm 13. \text{ For } \frac{dx}{dt} = 2 \text{ and } \frac{dy}{dt} = 3, \frac{dz}{dt} = \frac{1}{\pm 13} (5 \cdot 2 + 12 \cdot 3) = \pm \frac{46}{13}.$$

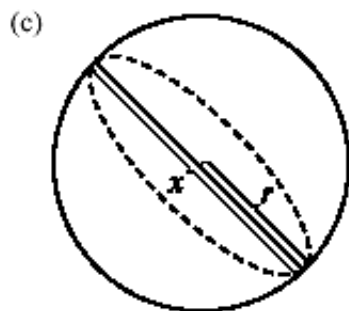
9. (a) Given: the rate of decrease of the surface area is $1 \text{ cm}^2/\text{min}$. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt = -1 \text{ cm}^2/\text{s}$.

- (b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10$ cm.

- (d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and

$$S = 4\pi r^2 = 4\pi \left(\frac{1}{2}x\right)^2 = \pi x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \frac{dx}{dt} = 2\pi x \frac{dx}{dt}.$$

- (e) $-1 = \frac{dS}{dt} = 2\pi x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{2\pi x}$. When $x = 10$, $\frac{dx}{dt} = -\frac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi} \text{ cm/min}$.



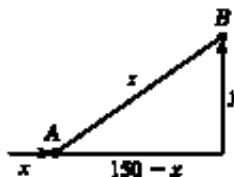
10. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35 \text{ km/h}$ and $dy/dt = 25 \text{ km/h}$.

- (b) Unknown: the rate at which the distance between the ships is changing at 4:00 P.M. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

$$(d) z^2 = (150 - x)^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2(150 - x) \left(-\frac{dx}{dt} \right) + 2y \frac{dy}{dt}$$

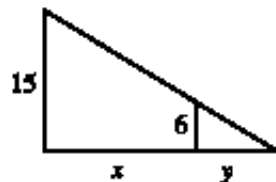
- (c) At 4:00 P.M., $x = 4(35) = 140$ and $y = 4(25) = 100 \Rightarrow z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10,100}$.

$$\text{So } \frac{dz}{dt} = \frac{1}{z} \left[(x - 150) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{-10(35) + 100(25)}{\sqrt{10,100}} = \frac{215}{\sqrt{101}} \approx 21.4 \text{ km/h}.$$



12. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\frac{d}{dt}(x + y)$ when $x = 40$ ft.

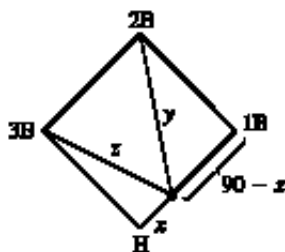


(d) By similar triangles, $\frac{15}{6} = \frac{x + y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x + y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{5}{3} \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

16. We are given that $\frac{dx}{dt} = 24$ ft/s.

(a)



$$y^2 = (90 - x)^2 + 90^2 \Rightarrow 2y \frac{dy}{dt} = 2(90 - x) \left(-\frac{dx}{dt}\right). \text{ When } x = 45,$$

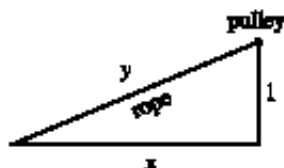
$$y = \sqrt{45^2 + 90^2} = 45\sqrt{5}, \text{ so } \frac{dy}{dt} = \frac{90 - x}{y} \left(-\frac{dx}{dt}\right) = \frac{45}{45\sqrt{5}}(-24) = -\frac{24}{\sqrt{5}},$$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$$z^2 = x^2 + 90^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt}. \text{ When } x = 45, z = 45\sqrt{5}, \text{ so } \frac{dz}{dt} = \frac{45}{45\sqrt{5}}(24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s.}$$

18.



$$\text{Given } \frac{dy}{dt} = -1 \text{ m/s, find } \frac{dx}{dt} \text{ when } x = 8 \text{ m. } y^2 = x^2 + 1 \Rightarrow 2y \frac{dy}{dt} = 2x \frac{dx}{dt} \Rightarrow$$

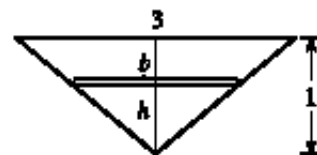
$$\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt} = -\frac{y}{x}. \text{ When } x = 8, y = \sqrt{65}, \text{ so } \frac{dx}{dt} = -\frac{\sqrt{65}}{8}. \text{ Thus, the boat}$$

approaches the dock at $\frac{\sqrt{65}}{8} \approx 1.01$ m/s.

23. By similar triangles, $\frac{3}{1} = \frac{b}{h}$, so $b = 3h$. The trough has volume

$$V = \frac{1}{2}bh(10) = 5(3h)h = 15h^2 \Rightarrow 12 = \frac{dV}{dt} = 30h \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{5h}.$$

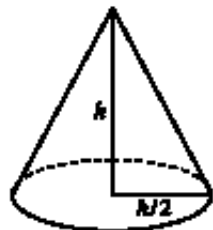
$$\text{When } h = \frac{1}{2}, \frac{dh}{dt} = \frac{2}{5 \cdot \frac{1}{2}} = \frac{4}{5} \text{ ft/min.}$$



25. We are given that $\frac{dV}{dt} = 30 \text{ ft}^3/\text{min}$. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12} \Rightarrow$

$$\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \Rightarrow 30 = \frac{\pi h^2}{4} \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{120}{\pi h^2}. \text{ When } h = 10 \text{ ft,}$$

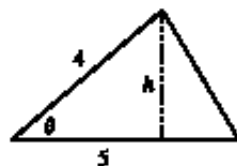
$$\frac{dh}{dt} = \frac{120}{10^2\pi} = \frac{6}{5\pi} \approx 0.38 \text{ ft/min.}$$



27. $A = \frac{1}{2}bh$, but $b = 5 \text{ m}$ and $\sin \theta = \frac{h}{4} \Rightarrow h = 4 \sin \theta$, so $A = \frac{1}{2}(5)(4 \sin \theta) = 10 \sin \theta$.

We are given $\frac{d\theta}{dt} = 0.06 \text{ rad/s}$, so $\frac{dA}{dt} = \frac{dA}{d\theta} \frac{d\theta}{dt} = (10 \cos \theta)(0.06) = 0.6 \cos \theta$.

When $\theta = \frac{\pi}{3}$, $\frac{dA}{dt} = 0.6(\cos \frac{\pi}{3}) = (0.6)(\frac{1}{2}) = 0.3 \text{ m}^2/\text{s}$.

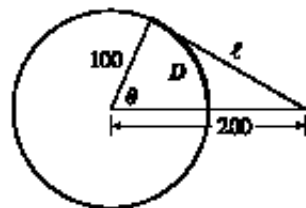


31. With $R_1 = 80$ and $R_2 = 100$, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80} + \frac{1}{100} = \frac{180}{8000} = \frac{9}{400}$, so $R = \frac{400}{9}$. Differentiating $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

with respect to t , we have $-\frac{1}{R^2} \frac{dR}{dt} = -\frac{1}{R_1^2} \frac{dR_1}{dt} - \frac{1}{R_2^2} \frac{dR_2}{dt} \Rightarrow \frac{dR}{dt} = R^2 \left(\frac{1}{R_1^2} \frac{dR_1}{dt} + \frac{1}{R_2^2} \frac{dR_2}{dt} \right)$. When $R_1 = 80$ and

$$R_2 = 100, \frac{dR}{dt} = \frac{400^2}{9^2} \left[\frac{1}{80^2} (0.3) + \frac{1}{100^2} (0.2) \right] = \frac{107}{810} \approx 0.132 \Omega/\text{s}.$$

37. Let the distance between the runner and the friend be ℓ . Then by the Law of Cosines,
 $\ell^2 = 200^2 + 100^2 - 2 \cdot 200 \cdot 100 \cdot \cos \theta = 50,000 - 40,000 \cos \theta$ (*). Differentiating
 implicitly with respect to t , we obtain $2\ell \frac{d\ell}{dt} = -40,000(-\sin \theta) \frac{d\theta}{dt}$. Now if D is the
 distance run when the angle is θ radians, then by the formula for the length of an arc



on a circle, $s = r\theta$, we have $D = 100\theta$, so $\theta = \frac{1}{100}D \Rightarrow \frac{d\theta}{dt} = \frac{1}{100} \frac{dD}{dt} = \frac{7}{100}$. To substitute into the expression for

$\frac{d\ell}{dt}$, we must know $\sin \theta$ at the time when $\ell = 200$, which we find from (*): $200^2 = 50,000 - 40,000 \cos \theta \Leftrightarrow$

$$\cos \theta = \frac{1}{4} \Rightarrow \sin \theta = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}. \text{ Substituting, we get } 2(200) \frac{d\ell}{dt} = 40,000 \frac{\sqrt{15}}{4} \left(\frac{7}{100} \right) \Rightarrow$$

$d\ell/dt = \frac{7\sqrt{15}}{4} \approx 6.78 \text{ m/s}$. Whether the distance between them is increasing or decreasing depends on the direction in which the runner is running.