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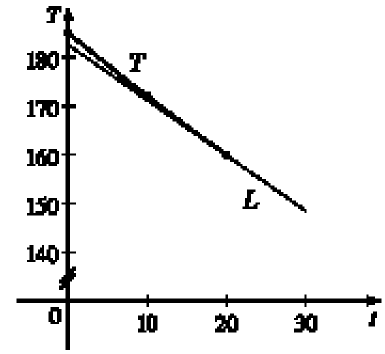
1. As in Example 1, $T(0) = 185$, $T(10) = 172$, $T(20) = 160$, and

$$T'(20) \approx \frac{T(10) - T(20)}{10 - 20} = \frac{172 - 160}{-10} = -1.2°\text{F/min.}$$

$T(30) \approx T(20) + T'(20)(30 - 20) \approx 160 - 1.2(10) = 148°\text{F}$.



We would expect the temperature of the turkey to get closer to 75°F as time increases. Since the temperature decreased 13°F in the first 10 minutes and 12°F in the second 10 minutes, we can assume that the slopes of the tangent line are increasing through negative values: $-1.3, -1.2, \ldots$. Hence, the tangent lines are under the curve and 148°F is an underestimate. From the figure, we estimate the slope of the tangent line at $t = 20$ to be $\frac{184 - 147}{0 - 30} = -\frac{37}{30}$. Then the linear approximation becomes $T(30) \approx T(20) + T'(20) \cdot 10 \approx 160 - \frac{37}{30}(10) = 147\frac{2}{3} \approx 147.7$.

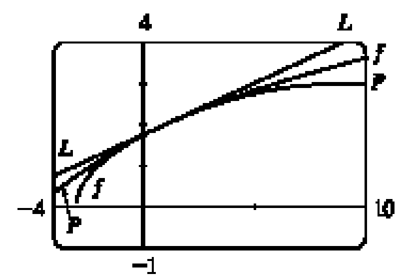
3. If $P(x) = A + B(x - a) + C(x - a)^2$, then $P'(x) = B + 2C(x - a)$ and $P''(x) = 2C$. Applying the conditions (i), (ii), and (iii), we get

$$\begin{aligned} P(a) &= f(a): & A &= f(a) \\ P'(a) &= f'(a): & B &= f'(a) \\ P''(a) &= f''(a): & 2C &= f''(a) \quad \Rightarrow \quad C = \tfrac{1}{2} f''(a) \end{aligned}$$

Thus, $P(x) = A + B(x - a) + C(x - a)^2$ can be written in the form $P(x) = f(a) + f'(a)(x - a) + \frac{1}{2} f''(a)(x - a)^2$.

4. From Example 2 in Section 3.8, we have $f(1) = 2$, $f'(1) = \frac{1}{4}$, and $f'(x) = \frac{1}{2}(x + 3)^{-1/2}$. So $f''(x) = -\frac{1}{4}(x + 3)^{-3/2} \quad \Rightarrow$ $f''(1) = -\frac{1}{32}$. From Problem 3, the quadratic approximation $P(x)$ is

$$\sqrt{x + 3} \approx f(1) + f'(1)(x - 1) + \tfrac{1}{2} f''(1)(x - 1)^2 = 2 + \tfrac{1}{4}(x - 1) - \tfrac{1}{64}(x - 1)^2.$$



The figure shows the function $f(x) = \sqrt{x + 3}$ together with its linear approximation $L(x) = \frac{1}{4}x + \frac{7}{4}$ and its quadratic approximation $P(x)$. You can see that $P(x)$ is a better approximation than $L(x)$ and this is borne out by the numerical values in the following chart.

	from $L(x)$	actual value	from $P(x)$
$\sqrt{3.98}$	1.9950	1.99499373 . . .	1.99499375
$\sqrt{4.05}$	2.0125	2.01246118 . . .	2.01246094
$\sqrt{4.2}$	2.0500	2.04939015 . . .	2.04937500

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5. $f(x) = x^4 + 3x^2 \quad \Rightarrow \quad f'(x) = 4x^3 + 6x$, so $f(-1) = 4$ and $f'(-1) = -10$.

Thus, $L(x) = f(-1) + f'(-1)(x - (-1)) = 4 + (-10)(x + 1) = -10x - 6$.

8. $f(x) = x^{3/4} \quad \Rightarrow \quad f'(x) = \frac{3}{4}x^{-1/4}$, so $f(16) = 8$ and $f'(16) = \frac{3}{8}$.

Thus, $L(x) = f(16) + f'(16)(x - 16) = 8 + \frac{3}{8}(x - 16) = \frac{3}{8}x + 2$.

15. To estimate $(2.001)^5$, we'll find the linearization of $f(x) = x^5$ at $a = 2$. Since $f'(x) = 5x^4$, $f(2) = 32$, and $f'(2) = 80$, we have $L(x) = 32 + 80(x - 2) = 80x - 128$. Thus, $x^5 \approx 80x - 128$ when x is near 2 , so

$(2.001)^5 \approx 80(2.001) - 128 = 160.08 - 128 = 32.08$.

21. $y = f(x) = \ln x \quad \Rightarrow \quad f'(x) = 1/x$, so $f(1) = 0$ and $f'(1) = 1$. The linear approximation of f at 1 is $f(1) + f'(1)(x - 1) = 0 + 1(x - 1) = x - 1$. Now $f(1.05) = \ln 1.05 \approx 1.05 - 1 = 0.05$, so the approximation is reasonable.

25. (a) $y = e^{x/10} \quad \Rightarrow \quad dy = e^{x/10} \cdot \frac{1}{10}\,dx = \frac{1}{10}e^{x/10}dx$

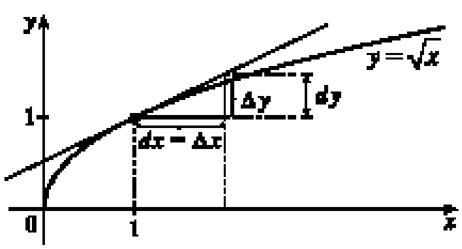
(b) $x = 0$ and $dx = 0.1 \quad \Rightarrow \quad dy = \frac{1}{10}e^{0/10}(0.1) = 0.01$.

$\Delta y = f(x + \Delta x) - f(x) = e^{(x+\Delta x)/10} - e^x = e^{(0+0.1)/10} - e^0 = e^{1/100} - 1 \approx 0.0101$

26. (a) $y = \sqrt{x} \quad \Rightarrow \quad dy = \frac{1}{2}x^{-1/2}dx = \dfrac{1}{2\sqrt{x}}\,dx$

(b) $x = 1$ and $dx = 1 \quad \Rightarrow \quad dy = \frac{1}{2(1)}(1) = \frac{1}{2}$. $\Delta y = f(x + \Delta x) - f(x) = \sqrt{1+1} - \sqrt{1} = \sqrt{2} - 1 \approx 0.414$.

(c)



Remember, Δy represents the amount that the curve $y = f(x)$ rises or falls when x changes by an amount dx, whereas dy represents the amount that the tangent line rises or falls (the change in the linearization).