

1. The differentiation formula for logarithmic functions,  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$ , is simplest when  $a = e$  because  $\ln e = 1$ .

$$2. f(x) = \ln(x^2 + 10) \Rightarrow f'(x) = \frac{1}{x^2 + 10} \frac{d}{dx}(x^2 + 10) = \frac{2x}{x^2 + 10}$$

$$7. f(x) = \log_2(1 - 3x) \Rightarrow f'(x) = \frac{1}{(1 - 3x) \ln 2} \frac{d}{dx}(1 - 3x) = \frac{-3}{(1 - 3x) \ln 2} \text{ or } \frac{3}{(3x - 1) \ln 2}$$

$$8. f(x) = \log_5(xe^x) \Rightarrow f'(x) = \frac{1}{xe^x \ln 5} \frac{d}{dx}(xe^x) = \frac{1}{xe^x \ln 5} (xe^x + e^x \cdot 1) = \frac{e^x(x+1)}{xe^x \ln 5} = \frac{x+1}{x \ln 5}$$

*Another solution:* We can change the form of the function by first using logarithm properties.

$$f(x) = \log_5(xe^x) = \log_5 x + \log_5 e^x \Rightarrow f'(x) = \frac{1}{x \ln 5} + \frac{1}{e^x \ln 5} \cdot e^x = \frac{1}{x \ln 5} + \frac{1}{\ln 5} \text{ or } \frac{1+x}{x \ln 5}$$

$$10. f(t) = \frac{1 + \ln t}{1 - \ln t} \Rightarrow f'(t) = \frac{(1 - \ln t)(1/t) - (1 + \ln t)(-1/t)}{(1 - \ln t)^2} = \frac{(1/t)[(1 - \ln t) + (1 + \ln t)]}{(1 - \ln t)^2} = \frac{2}{t(1 - \ln t)^2}$$

$$20. y = \frac{\ln x}{x^2} \Rightarrow y' = \frac{x^2(1/x) - (\ln x)(2x)}{(x^2)^2} = \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3} \Rightarrow$$

$$y'' = \frac{x^3(-2/x) - (1 - 2 \ln x)(3x^2)}{(x^3)^2} = \frac{x^2(-2 - 3 + 6 \ln x)}{x^6} = \frac{6 \ln x - 5}{x^4}$$

$$35. y = (\tan x)^{1/x} \Rightarrow \ln y = \ln(\tan x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln \tan x \Rightarrow$$

$$\frac{1}{y} y' = \frac{1}{x} \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln \tan x \cdot \left(-\frac{1}{x^2}\right) \Rightarrow y' = y \left( \frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2} \right) \Rightarrow$$

$$y' = (\tan x)^{1/x} \left( \frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2} \right) \text{ or } y' = (\tan x)^{1/x} \cdot \frac{1}{x} \left( \csc x \sec x - \frac{\ln \tan x}{x} \right)$$

$$36. y = (\sin x)^{\ln x} \Rightarrow \ln y = \ln(\sin x)^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln \sin x \Rightarrow$$

$$\frac{1}{y} y' = \ln x \cdot \frac{1}{\sin x} \cdot \cos x + \ln \sin x \cdot \frac{1}{x} \Rightarrow y' = y \left( \ln x \cdot \frac{\cos x}{\sin x} + \frac{\ln \sin x}{x} \right) \Rightarrow$$

$$y' = (\sin x)^{\ln x} \left( \ln x \cot x + \frac{\ln \sin x}{x} \right)$$

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**39.**  $f(x) = \ln(x-1) \Rightarrow f'(x) = 1/(x-1) = (x-1)^{-1} \Rightarrow f''(x) = -(x-1)^{-2} \Rightarrow f'''(x) = 2(x-1)^{-3} \Rightarrow$   
 $f^{(4)}(x) = -2 \cdot 3(x-1)^{-4} \Rightarrow \dots \Rightarrow f^{(n)}(x) = (-1)^{n-1} \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot (n-1)(x-1)^{-n} = (-1)^{n-1} \frac{(n-1)!}{(x-1)^n}$

**41.** If  $f(x) = \ln(1+x)$ , then  $f'(x) = \frac{1}{1+x}$ , so  $f'(0) = 1$ .

Thus,  $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = \lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = f'(0) = 1$ .