Name: ___________________ Class:___ Date: ______________

1. (a) $\frac{d}{dx}(xy + 2x + 3x^2) = \frac{d}{dx}(4) \Rightarrow (x \cdot y' + y \cdot 1) + 2 + 6x = 0 \Rightarrow xy' = -y - 2 - 6x \Rightarrow$
$y' = \frac{-y - 2 - 6x}{x}$ or $y' = -6 - \frac{y+2}{x}$.

(b) $xy + 2x + 3x^2 = 4 \Rightarrow xy = 4 - 2x - 3x^2 \Rightarrow y = \frac{4 - 2x - 3x^2}{x} = \frac{4}{x} - 2 - 3x$, so $y' = -\frac{4}{x^2} - 3$.

(c) From part (a), $y' = \frac{-y - 2 - 6x}{x} = \frac{-(4/x - 2 - 3x) - 2 - 6x}{x} = \frac{-4/x - 3x}{x} = -\frac{4}{x^2} - 3$.

4. $\frac{d}{dx}(x^2 - 2xy + y^3) = \frac{d}{dx}(c) \Rightarrow 2x - 2(xy' + y \cdot 1) + 3y^2y' = 0 \Rightarrow 2x - 2y = 2xy' - 3y^2y' \Rightarrow$
$2x - 2y = y'(2x - 3y^2) \Rightarrow y' = \frac{2x - 2y}{2x - 3y^2}$

8. $\frac{d}{dx}(1 + x) = \frac{d}{dx}[\sin(xy^2)] \Rightarrow 1 = [\cos(xy^2)](x \cdot 2yy' + y^2 \cdot 1) \Rightarrow 1 = 2xy\cos(xy^2)y' + y^2\cos(xy^2) \Rightarrow$
$1 - y^2\cos(xy^2) = 2xy\cos(xy^2)y' \Rightarrow y' = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$

16. $x^2 + 2xy - y^2 + x = 2 \Rightarrow 2x + 2(xy' + y \cdot 1) - 2yy' + 1 = 0 \Rightarrow 2xy' - 2yy' = -2x - 2y - 1 \Rightarrow$
$y'(2x - 2y) = -2x - 2y - 1 \Rightarrow y' = \frac{-2x - 2y - 1}{2x - 2y}$. When $x = 1$ and $y = 2$, we have $y' = \frac{-2 - 4 - 1}{2 - 4} = \frac{-7}{-2} = \frac{7}{2}$,
so an equation of the tangent line is $y - 2 = \frac{7}{2}(x - 1)$ or $y = \frac{7}{2}x - \frac{3}{2}$.

20. $y^2(y^2 - 4) = x^2(x^2 - 5) \Rightarrow y^4 - 4y^2 = x^4 - 5x^2 \Rightarrow 4y^3y' - 8yy' = 4x^3 - 10x$. When $x = 0$ and $y = -2$, we have $-32y' + 16y' = 0 \Rightarrow -16y' = 0 \Rightarrow y' = 0$, so an equation of the tangent line is $y + 2 = 0(x - 0)$ or $y = -2$.

29. $y = \tan^{-1}\sqrt{x} \Rightarrow y' = \frac{1}{1 + (\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x}) = \frac{1}{1 + x}\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2\sqrt{x}(1 + x)}$

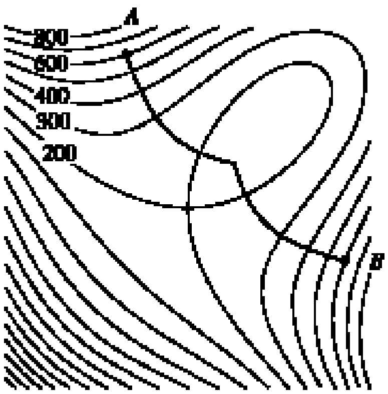
30. $y = \sqrt{\tan^{-1} x} = (\tan^{-1} x)^{1/2} \Rightarrow$
$y' = \frac{1}{2}(\tan^{-1} x)^{-1/2} \cdot \frac{d}{dx}(\tan^{-1} x) = \frac{1}{2\sqrt{\tan^{-1} x}} \cdot \frac{1}{1 + x^2} = \frac{1}{2\sqrt{\tan^{-1} x}(1 + x^2)}$

36. (a) Let $y = \cos^{-1} x$. Then $\cos y = x$ and $0 \le y \le \pi \;\Rightarrow\; \dfrac{d}{dx}(\cos y) = \dfrac{d}{dx}(x) \;\Rightarrow\; -\sin y \dfrac{dy}{dx} = 1 \;\Rightarrow$

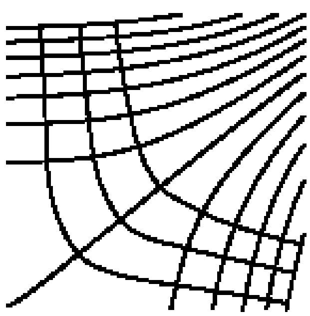
$$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$ (Note that $\sin y \ge 0$ for $0 \le y \le \pi$.)

(b) $y = x\cos^{-1} x - \sqrt{1-x^2} \;\Rightarrow\; y' = \cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}} + \dfrac{x}{\sqrt{1-x^2}} = \cos^{-1} x$

41.

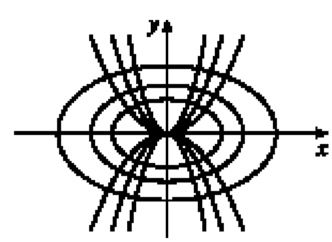


42. The orthogonal family represents the direction of the wind.



45. $y = cx^2 \;\Rightarrow\; y' = 2cx$ and $x^2 + 2y^2 = k \;\Rightarrow\; 2x + 4yy' = 0 \;\Rightarrow$

$2yy' = -x \;\Rightarrow\; y' = -\dfrac{x}{2(y)} = -\dfrac{x}{2(cx^2)} = -\dfrac{1}{2cx}$, so the curves are orthogonal.



53. (a) If $y = f^{-1}(x)$, then $f(y) = x$. Differentiating implicitly with respect to x and remembering that y is a function of x, we get $f'(y)\dfrac{dy}{dx} = 1$, so $\dfrac{dy}{dx} = \dfrac{1}{f'(y)} \;\Rightarrow\; (f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

(b) $f(4) = 5 \;\Rightarrow\; f^{-1}(5) = 4$. By part (a), $(f^{-1})'(5) = 1/f'(f^{-1}(5)) = 1/f'(4) = 1/\left(\frac{2}{3}\right) = \frac{3}{2}$.

54. (a) $f(x) = 2x + \cos x \quad \Rightarrow \quad f'(x) = 2 - \sin x > 0$ for all x. Thus, f is increasing for all x and is therefore one-to-one.

(b) Since f is one-to-one, $f^{-1}(1) = k \quad \Leftrightarrow \quad f(k) = 1$. By inspection, we see that $f(0) = 2(0) + \cos 0 = 1$, so $k = f^{-1}(1) = 0$.

(c) $\left(f^{-1}\right)'(1) = 1/f'\left(f^{-1}(1)\right) = 1/f'(0) = 1/(2 - \sin 0) = \frac{1}{2}$