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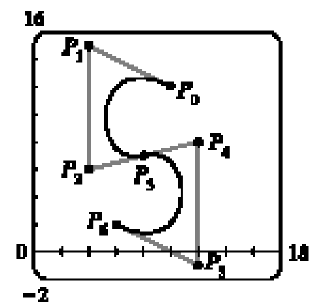
2. It suffices to show that the slope of the tangent at P_0 is the same as that of line segment P_0P_1, namely $\frac{y_1 - y_0}{x_1 - x_0}$. We calculate the slope of the tangent to the Bézier curve:

$$\frac{dy/dt}{dx/dt} = \frac{-3y_0(1-t)^2 + 3y_1\left[-2t(1-t) + (1-t)^2\right] + 3y_2\left[-t^2 + (2t)(1-t)\right] + 3y_3t^2}{-3x_0^2(1-t) + 3x_1[-2t(1-t) + (1-t)^2] + 3x_2[-t^2 + (2t)(1-t)] + 3x_3t^2}$$

At point P_0, $t = 0$, so the slope of the tangent is $\frac{-3y_0 + 3y_1}{-3x_0 + 3x_1} = \frac{y_1 - y_0}{x_1 - x_0}$. So the tangent to the curve at P_0 passes through P_1. Similarly, the slope of the tangent at point P_3 (where $t = 1$) is $\frac{-3y_2 + 3y_3}{-3x_2 + 3x_3} = \frac{y_3 - y_2}{x_3 - x_2}$, which is also the slope of line P_2P_3.

5. We use the same P_0 and P_1 as in Problem 4, and use part of our C as the top of an S. To prevent the center line from slanting up too much, we move P_2 up to $(4, 6)$ and P_3 down and to the left, to $(8, 7)$. In order to have a smooth joint between the top and bottom halves of the S (and a symmetric S), we determine points P_4, P_5, and P_6 by rotating points P_2, P_1, and P_0 about the center of the letter (point P_3). The points are therefore $P_4(12, 8)$, $P_5(12, -1)$, and $P_6(6, 2)$.



2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$. Then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$.

11. $y = \cos(a^3 + x^3) \quad \Rightarrow \quad y' = -\sin(a^3 + x^3) \cdot 3x^2 \quad [a^3 \text{ is just a constant}] \quad = -3x^2\sin(a^3 + x^3)$

12. $y = a^3 + \cos^3 x \quad \Rightarrow \quad y' = 3(\cos x)^2(-\sin x) \quad [a^3 \text{ is just a constant}] \quad = -3\sin x\cos^2 x$

14. $y = 3\cot(n\theta) \quad \Rightarrow \quad y' = 3[-\csc^2(n\theta)\cdot n] = -3n\csc^2(n\theta)$

16. $y = e^{-5x}\cos 3x \quad \Rightarrow \quad y' = e^{-5x}(-3\sin 3x) + (\cos 3x)(-5e^{-5x}) = -e^{-5x}(3\sin 3x + 5\cos 3x)$

24. $y = e^{k\tan\sqrt{x}} \quad \Rightarrow \quad y' = e^{k\tan\sqrt{x}}\cdot\frac{d}{dx}\left(k\tan\sqrt{x}\right) = e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}\right) = \frac{k\sec^2\sqrt{x}}{2\sqrt{x}}e^{k\tan\sqrt{x}}$

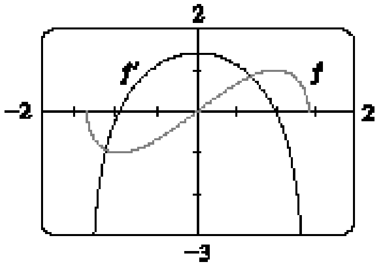
30. $y = \sin(\sin(\sin x)) \quad \Rightarrow \quad y' = \cos(\sin(\sin x))\frac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\,\cos(\sin x)\,\cos x$

36. $y = \sin x + \sin^2 x \quad \Rightarrow \quad y' = \cos x + 2\sin x \cos x$. At $(0, 0)$, $y' = 1$, and an equation of the tangent line is $y - 0 = 1(x - 0)$, or $y = x$.

39. (a) $f(x) = x\sqrt{2 - x^2} = x(2 - x^2)^{1/2} \quad \Rightarrow$

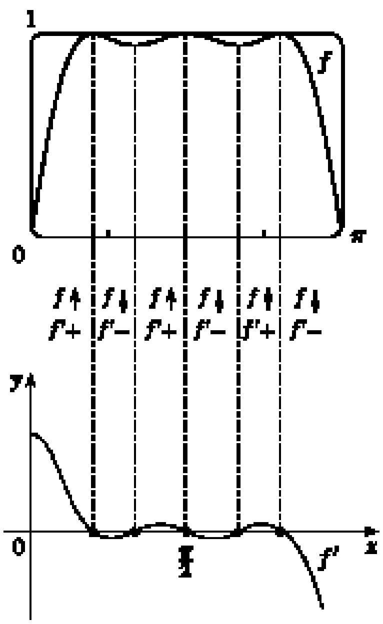
$$f'(x) = x \cdot \tfrac{1}{2}(2 - x^2)^{-1/2}(-2x) + (2 - x^2)^{1/2} \cdot 1 = (2 - x^2)^{-1/2}[-x^2 + (2 - x^2)] = \frac{2 - 2x^2}{\sqrt{2 - x^2}}$$

(b)



$f' = 0$ when f has a horizontal tangent line, f' is negative when f is decreasing, and f' is positive when f is increasing.

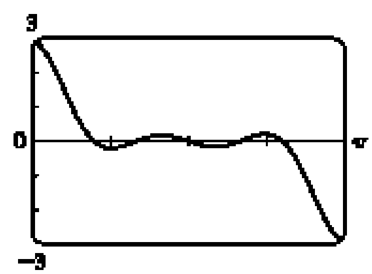
40. (a)



From the graph of f, we see that there are 5 horizontal tangents, so there must be 5 zeros on the graph of f'. From the symmetry of the graph of f, we must have the graph of f' as high at $x = 0$ as it is low at $x = \pi$. The intervals of increase and decrease as well as the signs of f' are indicated in the figure.

(b) $f(x) = \sin(x + \sin 2x) \quad \Rightarrow$

$$f'(x) = \cos(x + \sin 2x) \cdot \frac{d}{dx}(x + \sin 2x) = \cos(x + \sin 2x)(1 + 2\cos 2x)$$



41. $F(x) = f(g(x)) \quad \Rightarrow \quad F'(x) = f'(g(x)) \cdot g'(x)$, so $F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$

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43. (a) $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$, so $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30$.

(b) $H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x)$, so $H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36$.

52. $f(x) = xg(x^2) \Rightarrow f'(x) = xg'(x^2)\,2x + g(x^2) \cdot 1 = 2x^2g'(x^2) + g(x^2) \Rightarrow$
$f''(x) = 2x^2g''(x^2)\,2x + g'(x^2)\,4x + g'(x^2)\,2x = 4x^3g''(x^2) + 4xg'(x^2) + 2xg'(x^2) = 6xg'(x^2) + 4x^3g''(x^2)$

74. (a) $x = r(\theta - \sin\theta)$, $y = r\,(1 - \cos\theta) \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{r\,(\sin\theta)}{r\,(1-\cos\theta)} = \dfrac{\sin\theta}{1-\cos\theta}$. When $\theta = \frac{\pi}{3}$,

$\dfrac{dy}{dx} = \dfrac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = \sqrt{3}$, $(x, y) = \left(r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right), \frac{1}{2}r\right)$, and the tangent is $y - \frac{1}{2}r = \sqrt{3}\left[x - r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right]$.

(b) Horizontal tangent: $dy/dx = 0 \Leftrightarrow \sin\theta = 0$ (and $\cos\theta \neq 1$) $\Leftrightarrow \theta = (2n+1)\,\pi$. The corresponding points are $((2n+1)\,\pi r, 2r)$.

Vertical tangent: dy/dx is undefined $\Leftrightarrow 1 - \cos\theta = 0 \Leftrightarrow \cos\theta = 1 \Leftrightarrow \theta = 2n\pi$. The corresponding points are $(2n\pi r, 0)$.

(c)