Name: ___________________ Class:___ Date: _____________

**2.** $y = 2\csc x + 5\cos x \quad \Rightarrow \quad y' = -2\csc x \cot x - 5\sin x$

**7.** $y = \dfrac{x}{\cos x} \quad \Rightarrow \quad y' = \dfrac{(\cos x)(1) - (x)(-\sin x)}{(\cos x)^2} = \dfrac{\cos x + x\sin x}{\cos^2 x}$

**10.** $y = \dfrac{1 - \sec x}{\tan x} \quad \Rightarrow$

$$y' = \frac{\tan x\,(-\sec x \tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x\left(-\tan^2 x - \sec x + \sec^2 x\right)}{\tan^2 x} = \frac{\sec x\,(1 - \sec x)}{\tan^2 x}$$

**14.** $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \sec x \tan x$

**15.** $\dfrac{d}{dx}(\cot x) = \dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right) = \dfrac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = -\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x} = -\csc^2 x$

**18.** $y = e^x \cos x \quad \Rightarrow \quad y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x) \quad \Rightarrow$ the slope of the tangent line at $(0, 1)$ is $e^0(\cos 0 - \sin 0) = 1(1 - 0) = 1$ and an equation is $y - 1 = 1(x - 0)$ or $y = x + 1$.

**27.** $f(x) = x + 2\sin x$ has a horizontal tangent when $f'(x) = 0 \quad \Leftrightarrow \quad 1 + 2\cos x = 0 \quad \Leftrightarrow \quad \cos x = -\frac{1}{2} \quad \Leftrightarrow$ $x = \frac{2\pi}{3} + 2\pi n$ or $\frac{4\pi}{3} + 2\pi n$, where $n$ is an integer. Note that $\frac{4\pi}{3}$ and $\frac{2\pi}{3}$ are $\pm\frac{\pi}{3}$ units from $\pi$. This allows us to write the solutions in the more compact equivalent form $(2n + 1)\pi \pm \frac{\pi}{3}$, $n$ an integer.

**31.** (a) $x(t) = 8\sin t \quad \Rightarrow \quad v(t) = x'(t) = 8\cos t \quad \Rightarrow \quad a(t) = x''(t) = -8\sin t$

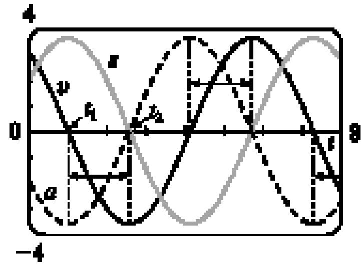
(b) The mass at time $t = \frac{2\pi}{3}$ has position $x\left(\frac{2\pi}{3}\right) = 8\sin\frac{2\pi}{3} = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$, velocity $v\left(\frac{2\pi}{3}\right) = 8\cos\frac{2\pi}{3} = 8\left(-\frac{1}{2}\right) = -4$, and acceleration $a\left(\frac{2\pi}{3}\right) = -8\sin\frac{2\pi}{3} = -8\left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3}$. Since $v\left(\frac{2\pi}{3}\right) < 0$, the particle is moving to the left. Because $v$ and $a$ have the same sign, the particle is speeding up.

**32.** (a) $s(t) = 2\cos t + 3\sin t \quad \Rightarrow$

$v(t) = -2\sin t + 3\cos t \quad \Rightarrow$

$a(t) = -2\cos t - 3\sin t$

(b)




(c) $s = 0 \quad \Rightarrow \quad t_2 \approx 2.55$. So the mass passes through the equilibrium position for the first time when $t \approx 2.55$ s.

(d) $v = 0 \quad \Rightarrow \quad t_1 \approx 0.98$, $s(t_1) \approx 3.61$ cm. So the mass travels a maximum of about 3.6 cm (upward and downward) from its equilibrium position.

(e) The speed $|v|$ is greatest when $s = 0$; that is, when $t = t_2 + n\pi$, $n$ a positive integer. The mass is speeding up when $v$ and $a$ have the same sign. From the figure, we see that this is the case on the intervals $(t_1 + n\pi, t_2 + n\pi)$ where $n$ is a whole number.

**39.** $$\lim_{t\to 0}\frac{\tan 6t}{\sin 2t} = \lim_{t\to 0}\left(\frac{\sin 6t}{t}\cdot\frac{1}{\cos 6t}\cdot\frac{t}{\sin 2t}\right) = \lim_{t\to 0}\frac{6\sin 6t}{6t}\cdot\lim_{t\to 0}\frac{1}{\cos 6t}\cdot\lim_{t\to 0}\frac{2t}{2\sin 2t}$$
$$= 6\lim_{t\to 0}\frac{\sin 6t}{6t}\cdot\lim_{t\to 0}\frac{1}{\cos 6t}\cdot\frac{1}{2}\lim_{t\to 0}\frac{2t}{\sin 2t} = 6(1)\cdot\frac{1}{1}\cdot\frac{1}{2}(1) = 3$$

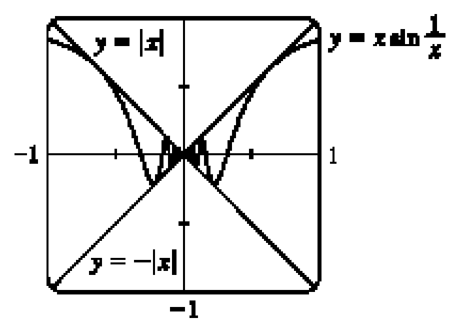
**41.** $$\lim_{\theta\to 0}\frac{\sin\theta}{\theta+\tan\theta} = \frac{\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}}{\lim\limits_{\theta\to 0}\dfrac{\theta+\tan\theta}{\theta}} = \frac{1}{\lim\limits_{\theta\to 0}\left(1+\dfrac{\sin\theta}{\theta}\cdot\dfrac{1}{\cos\theta}\right)} = \frac{1}{1+1\cdot 1} = \frac{1}{2}$$

**42.** (a) Let $\theta = \dfrac{1}{x}$. Then as $x \to \infty$, $\theta \to 0$, and $\lim\limits_{x\to\infty} x\sin\dfrac{1}{x} = \lim\limits_{\theta\to 0}\dfrac{1}{\theta}\sin\theta = \lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1$.

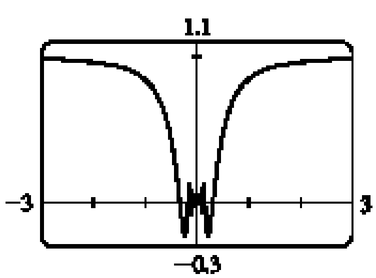
(b) Since $-1 \le \sin(1/x) \le 1$, we have (as illustrated in the figure)

$-|x| \le x\sin(1/x) \le |x|$. We know that $\lim\limits_{x\to 0}(|x|) = 0$ and

$\lim\limits_{x\to 0}(-|x|) = 0$; so by the Squeeze Theorem, $\lim\limits_{x\to 0} x\sin(1/x) = 0$.




(c)



**43.** By the definition of radian measure, $s = r\theta$, where $r$ is the radius of the circle.By drawing the bisector of the angle $\theta$, we can see that $\sin\dfrac{\theta}{2} = \dfrac{d/2}{r} \quad\Rightarrow\quad d = 2r\sin\dfrac{\theta}{2}$. So $\lim\limits_{\theta\to 0^+}\dfrac{s}{d} = \lim\limits_{\theta\to 0^+}\dfrac{r\theta}{2r\sin(\theta/2)} = \lim\limits_{\theta\to 0^+}\dfrac{2\cdot(\theta/2)}{2\sin(\theta/2)} = \lim\limits_{\theta\to 0}\dfrac{\theta/2}{\sin(\theta/2)} = 1$.
[This is just the reciprocal of the limit $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ combined with the fact that as $\theta \to 0$, $\frac{\theta}{2} \to 0$ also.]