Name: ___________________ Class:___ Date: _____________

4. By the Product Rule, $g(x) = \sqrt{x}\, e^x = x^{1/2} e^x \quad \Rightarrow \quad g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

7. $g(x) = \dfrac{3x-1}{2x+1} \overset{\text{QR}}{\Rightarrow} g'(x) = \dfrac{(2x+1)(3)-(3x-1)(2)}{(2x+1)^2} = \dfrac{6x+3-6x+2}{(2x+1)^2} = \dfrac{5}{(2x+1)^2}$

8. $f(t) = \dfrac{2t}{4+t^2} \overset{\text{QR}}{\Rightarrow} f'(t) = \dfrac{(4+t^2)(2)-(2t)(2t)}{(4+t^2)^2} = \dfrac{8+2t^2-4t^2}{(4+t^2)^2} = \dfrac{8-2t^2}{(4+t^2)^2}$

10. $R(t) = (t+e^t)(3-\sqrt{t}) \overset{\text{PR}}{\Rightarrow}$

$$R'(t) = (t+e^t)\left(-\tfrac{1}{2}t^{-1/2}\right) + (3-\sqrt{t})(1+e^t)$$
$$= \left(-\tfrac{1}{2}t^{1/2} - \tfrac{1}{2}t^{-1/2}e^t\right) + (3+3e^t-\sqrt{t}-\sqrt{t}\,e^t) = 3+3e^t-\tfrac{3}{2}\sqrt{t}-\sqrt{t}\,e^t - e^t/(2\sqrt{t})$$

13. $y = (r^2-2r)e^r \overset{\text{PR}}{\Rightarrow} y' = (r^2-2r)(e^r) + e^r(2r-2) = e^r(r^2-2r+2r-2) = e^r(r^2-2)$

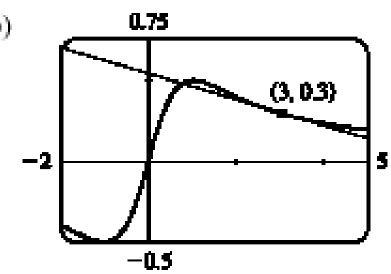
16. $z = w^{3/2}(w+ce^w) = w^{5/2} + cw^{3/2}e^w \quad \Rightarrow \quad z' = \frac{5}{2}w^{3/2} + c\left(w^{3/2}\cdot e^w + e^w \cdot \frac{3}{2}w^{1/2}\right) = \frac{5}{2}w^{3/2} + \frac{1}{2}cw^{1/2}e^w(2w+3)$

22. $y = \dfrac{\sqrt{x}}{x+1} \quad \Rightarrow \quad y' = \dfrac{(x+1)\left(\dfrac{1}{2\sqrt{x}}\right) - \sqrt{x}\,(1)}{(x+1)^2} = \dfrac{(x+1)-(2x)}{2\sqrt{x}\,(x+1)^2} = \dfrac{1-x}{2\sqrt{x}\,(x+1)^2}$. At $(4, 0.4)$, $y' = \frac{-3}{100} = -0.03$, and an equation of the tangent line is $y - 0.4 = -0.03(x-4)$, or $y = -0.03x + 0.52$. The slope of the normal line is $\frac{100}{3}$, so an equation of the normal line is $y - 0.4 = \frac{100}{3}(x-4) \quad \Leftrightarrow \quad y = \frac{100}{3}x - \frac{400}{3} + \frac{2}{5} \quad \Leftrightarrow \quad y = \frac{100}{3}x - \frac{1994}{15}$.

24. (a) $y = f(x) = \dfrac{x}{1+x^2} \quad \Rightarrow$

$f'(x) = \dfrac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x-3)$ or $y = -0.08x + 0.54$.

(b)



33. $f(x) = e^x g(x) \quad \Rightarrow \quad f'(x) = e^x g'(x) + g(x)e^x = e^x[g'(x) + g(x)]$. $f'(0) = e^0[g'(0)+g(0)] = 1(5+2) = 7$

37. (a) $y = xg(x) \quad \Rightarrow \quad y' = xg'(x) + g(x) \cdot 1 = xg'(x) + g(x)$

(b) $y = \dfrac{x}{g(x)} \quad \Rightarrow \quad y' = \dfrac{g(x) \cdot 1 - xg'(x)}{[g(x)]^2} = \dfrac{g(x) - xg'(x)}{[g(x)]^2}$

(c) $y = \dfrac{g(x)}{x} \quad \Rightarrow \quad y' = \dfrac{xg'(x) - g(x) \cdot 1}{(x)^2} = \dfrac{xg'(x) - g(x)}{x^2}$

39. If $P(t)$ denotes the population at time t and $A(t)$ the average annual income, then $T(t) = P(t)A(t)$ is the total personal income. The rate at which $T(t)$ is rising is given by $T'(t) = P(t)A'(t) + A(t)P'(t) \quad \Rightarrow$

$$T'(1999) = P(1999)A'(1999) + A(1999)P'(1999) = (961{,}400)(\$1400/\text{yr}) + (\$30{,}593)(9200/\text{yr})$$
$$= \$1{,}345{,}960{,}000/\text{yr} + \$281{,}455{,}600/\text{yr} = \$1{,}627{,}415{,}600/\text{yr}$$

So the total personal income was rising by about \$1.627 billion per year in 1999.

The term $P(t)A'(t) \approx \$1.346$ billion represents the portion of the rate of change of total income due to the existing population's increasing income. The term $A(t)P'(t) \approx \$281$ million represents the portion of the rate of change of total income due to increasing population.

41. f is increasing when f' is positive. $f(x) = x^3e^x \quad \Rightarrow \quad f'(x) = x^3e^x + e^x(3x^2) = x^2e^x(x+3)$. Now $x^2 \geq 0$ and $e^x > 0$ for all x, so $f'(x) > 0$ when $x + 3 > 0$ and $x \neq 0$; that is, when $x \in (-3, 0) \cup (0, \infty)$. So f is increasing on $(-3, \infty)$.

42. f is concave downward when f'' is negative. $f(x) = x^2e^x \quad \Rightarrow \quad f'(x) = x^2e^x + e^x(2x) \quad \Rightarrow$
$f''(x) = x^2e^x + e^x(2x) + e^x(2) + (2x)e^x = e^x(x^2 + 2x + 2 + 2x) = e^x(x^2 + 4x + 2)$. Note that $e^x > 0$ for all x and $f''(x) = 0 \quad \Leftrightarrow \quad x = -2 \pm \sqrt{2}$. $f''(x) < 0$ when $x \in \left(-2 - \sqrt{2}, -2 + \sqrt{2}\right)$.

46. (a) We use the Product Rule repeatedly: $F = fg \quad \Rightarrow \quad F' = f'g + fg' \quad \Rightarrow$
$F'' = (f''g + f'g') + (f'g' + fg'') = f''g + 2f'g' + fg''$.

(b) $F''' = f'''g + f''g' + 2\,(f''g' + f'g'') + f'g'' + fg''' = f'''g + 3f''g' + 3f'g'' + fg''' \quad \Rightarrow$
$$F^{(4)} = f^{(4)}g + f'''g' + 3\,(f'''g' + f''g'') + 3\,(f''g'' + f'g''') + f'g''' + fg^{(4)}$$
$$= f^{(4)}g + 4f'''g' + 6f''g'' + 4f'g''' + fg^{(4)}$$

(c) By analogy with the Binomial Theorem, we make the guess:

$$F^{(n)} = f^{(n)}g + nf^{(n-1)}g' + \binom{n}{2}f^{(n-2)}g'' + \cdots + \binom{n}{k}f^{(n-k)}g^{(k)} + \cdots + nf'g^{(n-1)} + fg^{(n)},$$

where $\dbinom{n}{k} = \dfrac{n!}{k!\,(n-k)!} = \dfrac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$.