

1. (a) e is the number such that $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$.

(b)

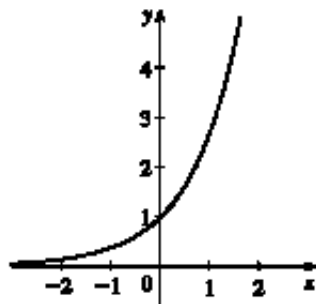
x	$\frac{2.7^x - 1}{x}$	x	$\frac{2.8^x - 1}{x}$
-0.001	0.9928	-0.001	1.0291
-0.0001	0.9932	-0.0001	1.0296
0.001	0.9937	0.001	1.0301
0.0001	0.9933	0.0001	1.0297

From the tables (to two decimal places),

$$\lim_{h \rightarrow 0} \frac{2.7^h - 1}{h} = 0.99 \text{ and } \lim_{h \rightarrow 0} \frac{2.8^h - 1}{h} = 1.03.$$

Since $0.99 < 1 < 1.03$, $2.7 < e < 2.8$.

2. (a)



The function value at $x = 0$ is 1 and the slope at $x = 0$ is 1.

(b) $f(x) = e^x$ is an exponential function and $g(x) = x^e$ is a power function. $\frac{d}{dx}(e^x) = e^x$ and $\frac{d}{dx}(x^e) = ex^{e-1}$.

(c) $s = f(t) = t^3 - 12t^2 + 36t, t \geq 0 \Rightarrow v(t) = f'(t) = 3t^2 - 24t + 36. a(t) = v'(t) = 6t - 24.$
 $a(3) = 6(3) - 24 = -6$ (m/s)/s or m/s^2 .

3. $f(x) = 186.5$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

$$11. G(x) = \sqrt{x} - 2e^x = x^{1/2} - 2e^x \Rightarrow G'(x) = \frac{1}{2}x^{-1/2} - 2e^x = \frac{1}{2\sqrt{x}} - 2e^x$$

$$16. y = \sqrt{x}(x-1) = x^{3/2} - x^{1/2} \Rightarrow y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x-1) \quad [\text{factor out } \frac{1}{2}x^{-1/2}]$$

or $y' = \frac{3x-1}{2\sqrt{x}}$.

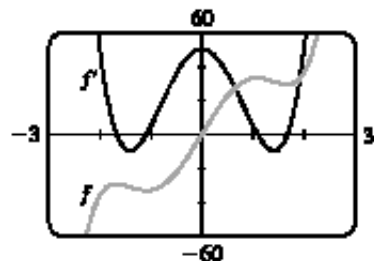
$$21. v = t^2 - \frac{1}{\sqrt[4]{t^3}} = t^2 - t^{-3/4} \Rightarrow v' = 2t - \left(-\frac{3}{4}\right)t^{-7/4} = 2t + \frac{3}{4t^{7/4}} = 2t + \frac{3}{4t\sqrt[4]{t^3}}$$

$$23. z = \frac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y \Rightarrow z' = -10Ay^{-11} + Be^y = -\frac{10A}{y^{11}} + Be^y$$

26. $y = (1 + 2x)^2 = 1 + 4x + 4x^2 \Rightarrow y' = 4 + 8x$. At $(1, 9)$, $y' = 12$ and an equation of the tangent line is $y - 9 = 12(x - 1)$ or $y = 12x - 3$. The slope of the normal line is $-\frac{1}{12}$ (the negative reciprocal of 12) and an equation of the normal line is $y - 9 = -\frac{1}{12}(x - 1)$ or $y = -\frac{1}{12}x + \frac{109}{12}$.

30. $f(x) = 3x^5 - 20x^3 + 50x \Rightarrow f'(x) = 15x^4 - 60x^2 + 50$.

Notice that $f'(x) = 0$ when f has a horizontal tangent and that f' is an even function while f is an odd function.

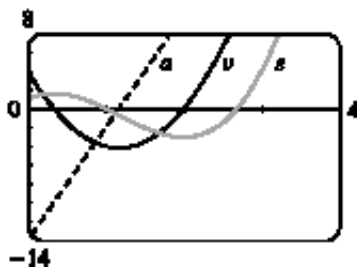


38. $G(r) = \sqrt{r} + \sqrt[3]{r} \Rightarrow G'(r) = \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3} \Rightarrow G''(r) = -\frac{1}{4}r^{-3/2} - \frac{2}{9}r^{-5/3}$

42. (a) $s = 2t^3 - 7t^2 + 4t + 1 \Rightarrow v(t) = s'(t) = 6t^2 - 14t + 4 \Rightarrow a(t) = v'(t) = 12t - 14$

(b) $a(1) = 12 - 14 = -2 \text{ m/s}^2$

(c)



58. $y = Ax^2 + Bx + C \Rightarrow y' = 2Ax + B \Rightarrow y'' = 2A$. We substitute these expressions into the equation $y'' + y' - 2y = x^2$ to get

$$(2A) + (2Ax + B) - 2(Ax^2 + Bx + C) = x^2$$

$$2A + 2Ax + B - 2Ax^2 - 2Bx - 2C = x^2$$

$$(-2A)x^2 + (2A - 2B)x + (2A + B - 2C) = (1)x^2 + (0)x + (0)$$

The coefficients of x^2 on each side must be equal, so $-2A = 1 \Rightarrow A = -\frac{1}{2}$. Similarly, $2A - 2B = 0 \Rightarrow A = B = -\frac{1}{2}$ and $2A + B - 2C = 0 \Rightarrow -1 - \frac{1}{2} - 2C = 0 \Rightarrow C = -\frac{3}{4}$.

63. $y = f(x) = ax^3 + bx^2 + cx + d \Rightarrow f'(x) = 3ax^2 + 2bx + c$. The point $(-2, 6)$ is on f , so $f(-2) = 6 \Rightarrow -8a + 4b - 2c + d = 6$ **(1)**. The point $(2, 0)$ is on f , so $f(2) = 0 \Rightarrow 8a + 4b + 2c + d = 0$ **(2)**. Since there are horizontal tangents at $(-2, 6)$ and $(2, 0)$, $f'(\pm 2) = 0$. $f'(-2) = 0 \Rightarrow 12a - 4b + c = 0$ **(3)** and $f'(2) = 0 \Rightarrow 12a + 4b + c = 0$ **(4)**. Subtracting equation **(3)** from **(4)** gives $8b = 0 \Rightarrow b = 0$. Adding **(1)** and **(2)** gives $8b + 2d = 6$, so $d = 3$ since $b = 0$. From **(3)** we have $c = -12a$, so **(2)** becomes $8a + 4(0) + 2(-12a) + 3 = 0 \Rightarrow 3 = 16a \Rightarrow a = \frac{3}{16}$. Now $c = -12a = -12\left(\frac{3}{16}\right) = -\frac{9}{4}$ and the desired cubic function is $y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$.