

The line from $P(2, f(2))$ to $Q(2+h, f(2+h))$ is the line that has slope $\frac{f(2+h) - f(2)}{h}$.

2. As h decreases, the line PQ becomes steeper, so its slope increases. So

$$0 < \frac{f(4) - f(2)}{4 - 2} < \frac{f(3) - f(2)}{3 - 2} < \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}. \text{ Thus, } 0 < \frac{1}{2} [f(4) - f(2)] < f(3) - f(2) < f'(2).$$

3. $g'(0)$ is the only negative value. The slope at $x = 4$ is smaller than the slope at $x = 2$ and both are smaller than the slope at $x = -2$. Thus, $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$.

4. (a) Since $g(5) = -3$, the point $(5, -3)$ is on the graph of g . Since $g'(5) = 4$, the slope of the tangent line at $x = 5$ is 4.

Using the point-slope form of a line gives us $y - (-3) = 4(x - 5)$, or $y = 4x - 23$.

(b) Since $(4, 3)$ is on $y = f(x)$, $f(4) = 3$. The slope of the tangent line between $(0, 2)$ and $(4, 3)$ is $\frac{1}{4}$, so $f'(4) = \frac{1}{4}$.

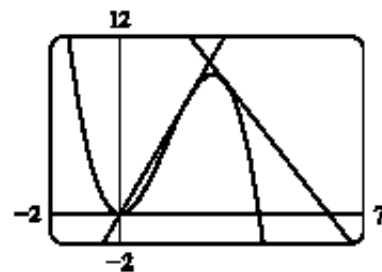
10. (a) Using Definition 2 with $G(x) = 4x^2 - x^3$, we have

$$\begin{aligned} G'(a) &= \lim_{h \rightarrow 0} \frac{G(a+h) - G(a)}{h} = \lim_{h \rightarrow 0} \frac{[4(a+h)^2 - (a+h)^3] - (4a^2 - a^3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4a^2 + 8ah + 4h^2 - (a^3 + 3a^2h + 3ah^2 + h^3) - 4a^2 + a^3}{h} = \lim_{h \rightarrow 0} \frac{8ah + 4h^2 - 3a^2h - 3ah^2 - h^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(8a + 4h - 3a^2 - 3ah - h^2)}{h} = \lim_{h \rightarrow 0} (8a + 4h - 3a^2 - 3ah - h^2) = 8a - 3a^2 \end{aligned}$$

(b) At the point $(2, 8)$, $G'(2) = 16 - 12 = 4$, and an equation of the tangent line

is $y - 8 = 4(x - 2)$, or $y = 4x$. At the point $(3, 9)$, $G'(3) = 24 - 27 = -3$,

and an equation of the tangent line is $y - 9 = -3(x - 3)$, or $y = -3x + 18$.



17. Use Definition 2 with $f(x) = 1/\sqrt{x+2}$.

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{(a+h)+2}} - \frac{1}{\sqrt{a+2}}}{h} = \lim_{h \rightarrow 0} \frac{\frac{\sqrt{a+2} - \sqrt{a+h+2}}{\sqrt{a+h+2}\sqrt{a+2}}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sqrt{a+2} - \sqrt{a+h+2}}{h\sqrt{a+h+2}\sqrt{a+2}} \cdot \frac{\sqrt{a+2} + \sqrt{a+h+2}}{\sqrt{a+2} + \sqrt{a+h+2}} \right] = \lim_{h \rightarrow 0} \frac{(a+2) - (a+h+2)}{h\sqrt{a+h+2}\sqrt{a+2}(\sqrt{a+2} + \sqrt{a+h+2})} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h\sqrt{a+h+2}\sqrt{a+2}(\sqrt{a+2} + \sqrt{a+h+2})} = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{a+h+2}\sqrt{a+2}(\sqrt{a+2} + \sqrt{a+h+2})} \\ &= \frac{-1}{(\sqrt{a+2})^2(2\sqrt{a+2})} = -\frac{1}{2(a+2)^{3/2}} \end{aligned}$$

Note that the answers to Exercises 19–24 are not unique.

19. By Definition 2, $\lim_{h \rightarrow 0} \frac{(1+h)^{10} - 1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 2, $\lim_{h \rightarrow 0} \frac{(1+h)^{10} - 1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

$$\begin{aligned} 25. v(5) &= f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0} \frac{[100 + 50(5+h) - 4.9(5+h)^2] - [100 + 50(5) - 4.9(5)^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{(100 + 250 + 50h - 4.9h^2 - 49h - 122.5) - (100 + 250 - 122.5)}{h} = \lim_{h \rightarrow 0} \frac{-4.9h^2 + h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-4.9h + 1)}{h} = \lim_{h \rightarrow 0} (-4.9h + 1) = 1 \text{ m/s} \end{aligned}$$

The speed when $t = 5$ is $|1| = 1$ m/s.

28. (a) $f'(5)$ is the rate of growth of the bacteria population when $t = 5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

29. (a) $f'(v)$ is the rate at which the fuel consumption is changing with respect to the speed. Its units are (gal/h)/(mi/h).

(b) The fuel consumption is decreasing by 0.05 (gal/h)/(mi/h) as the car's speed reaches 20 mi/h. So if you increase your speed to 21 mi/h, you could expect to decrease your fuel consumption by about 0.05 (gal/h)/(mi/h).

31. $T'(10)$ is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of $T'(10)$, we will average the difference quotients obtained using the times $t = 8$ and $t = 12$. Let $A = \frac{T(8) - T(10)}{8 - 10} = \frac{72 - 81}{-2} = 4.5$ and

$$B = \frac{T(12) - T(10)}{12 - 10} = \frac{88 - 81}{2} = 3.5. \text{ Then } T'(10) = \lim_{t \rightarrow 10} \frac{T(t) - T(10)}{t - 10} \approx \frac{A + B}{2} = \frac{4.5 + 3.5}{2} = 4 \text{ }^\circ\text{F/h.}$$

36. Since $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin(1/h) - 0}{h} = \lim_{h \rightarrow 0} h \sin(1/h). \text{ Since } -1 \leq \sin \frac{1}{h} \leq 1, \text{ we have}$$

$-|h| \leq |h| \sin \frac{1}{h} \leq |h| \Rightarrow -|h| \leq h \sin \frac{1}{h} \leq |h|$. Because $\lim_{h \rightarrow 0} (-|h|) = 0$ and $\lim_{h \rightarrow 0} |h| = 0$, we know that

$$\lim_{h \rightarrow 0} \left(h \sin \frac{1}{h} \right) = 0 \text{ by the Squeeze Theorem. Thus, } f'(0) = 0.$$