

1. (a) This is just the slope of the line through two points: $m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(3)}{x - 3}$.

(b) This is the limit of the slope of the secant line PQ as Q approaches P : $m = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$.

2. (a) Average velocity = $\frac{\Delta s}{\Delta t} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}$

(b) Instantaneous velocity = $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

3. The slope at D is the largest positive slope, followed by the positive slope at E . The slope at C is zero. The slope at B is steeper than at A (both are negative). In decreasing order, we have the slopes at: $D, E, C, A,$ and B .

5. (a) (i) Using Definition 1,

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow -3} \frac{f(x) - f(-3)}{x - (-3)} = \lim_{x \rightarrow -3} \frac{(x^2 + 2x) - (3)}{x - (-3)} = \lim_{x \rightarrow -3} \frac{(x+3)(x-1)}{x+3}$$

$$= \lim_{x \rightarrow -3} (x-1) = -4$$

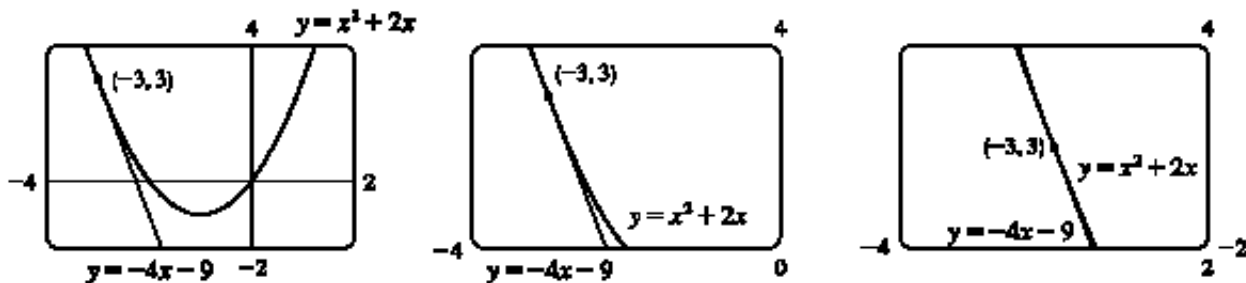
(ii) Using Equation 2,

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \rightarrow 0} \frac{[(-3+h)^2 + 2(-3+h)] - (3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{9 - 6h + h^2 - 6 + 2h - 3}{h} = \lim_{h \rightarrow 0} \frac{h(h-4)}{h} = \lim_{h \rightarrow 0} (h-4) = -4$$

(b) Using the point-slope form of the equation of a line, an equation of the tangent line is $y - 3 = -4(x + 3)$. Solving for y gives us $y = -4x - 9$, which is the slope-intercept form of the equation of the tangent line.

(c)



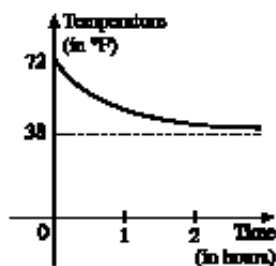
10. Using (1), $m = \lim_{x \rightarrow 0} \frac{\frac{2x}{(x+1)^2} - 0}{x - 0} = \lim_{x \rightarrow 0} \frac{2x}{x(x+1)^2} = \lim_{x \rightarrow 0} \frac{2}{(x+1)^2} = \frac{2}{1^2} = 2$.

Tangent line: $y - 0 = 2(x - 0) \Leftrightarrow y = 2x$

14. Let a denote the distance traveled from 1:00 to 1:02, b from 1:28 to 1:30, and c from 3:30 to 3:33, where all the times are relative to $t = 0$ at the beginning of the trip.

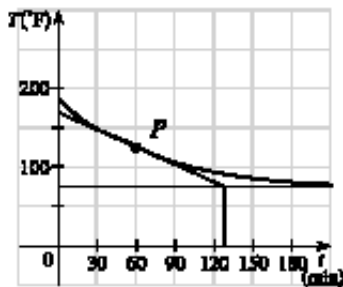


15. (a) Since the slope of the tangent at $t = 0$ is 0, the car's initial velocity was 0.
 (b) The slope of the tangent is greater at C than at B , so the car was going faster at C .
 (c) Near A , the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up. Near B , the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is the one at C , so at C the car had just finished speeding up, and was about to start slowing down.
 (d) Between D and E , the slope of the tangent is 0, so the car did not move during that time.
21. The sketch shows the graph for a room temperature of 72° and a refrigerator temperature of 38° . The initial rate of change is greater in magnitude than the rate of change after an hour.



22. The slope of the tangent (that is, the rate of change of temperature with respect to time) at $t = 1$ h seems to

be about $\frac{75 - 168}{132 - 0} \approx -0.7^\circ\text{F}/\text{min}$.



24. (a) (i) [1996, 2000]: $\frac{P(2000) - P(1996)}{2000 - 1996} = \frac{30,791 - 29,672}{4} = \frac{1119}{4} = 279.75$ thousand/year

(ii) [1998, 2000]: $\frac{P(2000) - P(1998)}{2000 - 1998} = \frac{30,791 - 30,248}{2} = \frac{543}{2} = 271.5$ thousand/year

(iii) [2000, 2002]: $\frac{P(2002) - P(2000)}{2002 - 2000} = \frac{31,414 - 30,791}{2} = \frac{623}{2} = 311.5$ thousand/year

(b) Using the values from (ii) and (iii), we have

$$\frac{271.5 + 311.5}{2} = 291.5 \text{ thousand/year.}$$

(c) Estimating A as (1998, 30,230) and B as (2002, 31,350), the slope at 2000

is $\frac{31,350 - 30,230}{2002 - 1998} = \frac{1120}{4} = 280$ thousand/year.

