Name: ___________________ Class:___ Date: ______________

1. (a) $\lim_{x\to a}[f(x)+h(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} h(x) = -3+8=5$

(b) $\lim_{x\to a}[f(x)]^2 = \left[\lim_{x\to a} f(x)\right]^2 = (-3)^2 = 9$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} = \sqrt[3]{\lim_{x\to a} h(x)} = \sqrt[3]{8} = 2$

(d) $\lim_{x\to a}\dfrac{1}{f(x)} = \dfrac{1}{\lim_{x\to a} f(x)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

(e) $\lim_{x\to a}\dfrac{f(x)}{h(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} h(x)} = \dfrac{-3}{8} = -\dfrac{3}{8}$

(f) $\lim_{x\to a}\dfrac{g(x)}{f(x)} = \dfrac{\lim_{x\to a} g(x)}{\lim_{x\to a} f(x)} = \dfrac{0}{-3} = 0$

(g) The limit does not exist, since $\lim_{x\to a} g(x) = 0$ but $\lim_{x\to a} f(x) \neq 0$.

(h) $\lim_{x\to a}\dfrac{2f(x)}{h(x)-f(x)} = \dfrac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)} = \dfrac{2(-3)}{8-(-3)} = -\dfrac{6}{11}$

2. (a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2+0=2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3 \cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

5.
$$\begin{aligned}
\lim_{x\to 8}(1+\sqrt[3]{x})(2-6x^2+x^3) &= \lim_{x\to 8}(1+\sqrt[3]{x}) \cdot \lim_{x\to 8}(2-6x^2+x^3) && \text{[Limit Law 4]}\\
&= \left(\lim_{x\to 8} 1 + \lim_{x\to 8}\sqrt[3]{x}\right)\cdot\left(\lim_{x\to 8} 2 - 6\lim_{x\to 8} x^2 + \lim_{x\to 8} x^3\right) && \text{[1, 2, and 3]}\\
&= (1+\sqrt[3]{8})\cdot(2-6\cdot 8^2+8^3) && \text{[7, 10, 9]}\\
&= (3)(130) = 390
\end{aligned}$$

8. (a) The left-hand side of the equation is not defined for $x=2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \to 2$, just as in Example 3. Remember that in finding $\lim_{x\to a} f(x)$, we never consider $x = a$.

9. $\lim_{x\to 2}\dfrac{x^2+x-6}{x-2} = \lim_{x\to 2}\dfrac{(x+3)(x-2)}{x-2} = \lim_{x\to 2}(x+3) = 2+3 = 5$

10. $\lim_{x\to -4}\dfrac{x^2+5x+4}{x^2+3x-4} = \lim_{x\to -4}\dfrac{(x+4)(x+1)}{(x+4)(x-1)} = \lim_{x\to -4}\dfrac{x+1}{x-1} = \dfrac{-4+1}{-4-1} = \dfrac{-3}{-5} = \dfrac{3}{5}$

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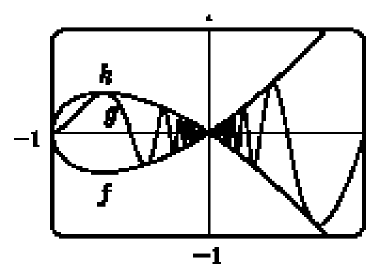
15. $\lim_{h\to 0}\frac{(4+h)^2-16}{h}=\lim_{h\to 0}\frac{(16+8h+h^2)-16}{h}=\lim_{h\to 0}\frac{8h+h^2}{h}=\lim_{h\to 0}\frac{h(8+h)}{h}=\lim_{h\to 0}(8+h)=8+0=8$

16. $$\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}=\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}=\lim_{h\to 0}\frac{(1+h)-1}{h\left(\sqrt{1+h}+1\right)}=\lim_{h\to 0}\frac{h}{h\left(\sqrt{1+h}+1\right)}$$
$$=\lim_{h\to 0}\frac{1}{\sqrt{1+h}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

17. By the formula for the sum of cubes, we have
$$\lim_{x\to -2}\frac{x+2}{x^3+8}=\lim_{x\to -2}\frac{x+2}{(x+2)(x^2-2x+4)}=\lim_{x\to -2}\frac{1}{x^2-2x+4}=\frac{1}{4+4+4}=\frac{1}{12}.$$

20. $$\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}=\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}=\lim_{h\to 0}\frac{3-(3+h)}{h(3+h)3}=\lim_{h\to 0}\frac{-h}{h(3+h)3}$$
$$=\lim_{h\to 0}\left[-\frac{1}{3(3+h)}\right]=-\frac{1}{\lim_{h\to 0}[3(3+h)]}=-\frac{1}{3(3+0)}=-\frac{1}{9}$$

26. Let $f(x)=-\sqrt{x^3+x^2}$, $g(x)=\sqrt{x^3+x^2}\,\sin(\pi/x)$, and $h(x)=\sqrt{x^3+x^2}$.
Then $-1\le\sin(\pi/x)\le 1 \;\Rightarrow$
$-\sqrt{x^3+x^2}\le\sqrt{x^3+x^2}\sin(\pi/x)\le\sqrt{x^3+x^2} \;\Rightarrow\; f(x)\le g(x)\le h(x)$. So
since $\lim_{x\to 0}f(x)=\lim_{x\to 0}h(x)=0$, by the Squeeze Theorem we have $\lim_{x\to 0}g(x)=0$.

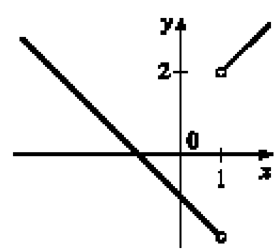


36. (a) (i) $\lim_{x\to 1^+}\frac{x^2-1}{|x-1|}=\lim_{x\to 1^+}\frac{x^2-1}{x-1}=\lim_{x\to 1^+}(x+1)=2$

(ii) $\lim_{x\to 1^-}\frac{x^2-1}{|x-1|}=\lim_{x\to 1^-}\frac{x^2-1}{-(x-1)}=\lim_{x\to 1^-}-(x+1)=-2$

(b) No, $\lim_{x\to 1}F(x)$ does not exist since $\lim_{x\to 1^+}F(x)\neq\lim_{x\to 1^-}F(x)$.

(c)



43. Let $f(x)=[\![x]\!]$ and $g(x)=-[\![x]\!]$. Then $\lim_{x\to 3}f(x)$ and $\lim_{x\to 3}g(x)$ do not exist [Example 9]
but $\lim_{x\to 3}[f(x)+g(x)]=\lim_{x\to 3}([\![x]\!]-[\![x]\!])=\lim_{x\to 3}0=0$.