Mathematics 1a, Section 5.5 Solutions

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4. Let $u = \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}dx$, so

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int \sin u(2du) = 2(-\cos u) + C = -2\cos\sqrt{x} + C$$

6. Let $u = \sin\theta$. Then $du = \cos\theta d\theta$, so

$$\int e^{\sin\theta}\cos\theta d\theta = \int e^u du = e^u + C = e^{\sin\theta} + C$$

16. Let $u = 1 - t^3$. Then $du = -3t^2 dt$, so

$$\int t^2\cos(1-t^3)dt = \int \cos u\left(-\frac{1}{3}du\right) = -\frac{1}{3}\sin u + C = -\frac{1}{3}\sin(1-t^3) + C$$

32. Let $u = x^2$. Then $du = 2xdx$, so

$$\int \frac{x}{1+x^4}dx = \int \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^2) + C$$

44. Let $u = 3x+1$, so $du = 3dx$. When $x = 1$, $u = 4$. When $x = 2$, $u = 7$. Thus

$$\int_1^2 \frac{dx}{3x+1} = \int_4^7 \frac{1}{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\left[\ln|u|\right]_4^7 = \frac{1}{3}(\ln 7 - \ln 4) = \frac{1}{3}\ln\frac{7}{4}$$

50. Let $u = a^2 - x^2$, so $du = -2xdx$. When $x = 0$, $u = a^2$, when $x = a$, $u = 0$. So

$$\int_0^a x\sqrt{a^2-x^2}dx = \int_{a^2}^0 u^{1/2}\left(-\frac{1}{2}u\right) = \frac{1}{2}\int_0^{a^2} u^{1/2}du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^{a^2} = \frac{1}{3}a^3$$

57. a. Let $u = \sqrt{x}$, so $du = \frac{1}{2\sqrt{x}}dx$ and $dx = 2\sqrt{x}du = 2udu$. When $x = 0$, $u = 0$, and when $x = 1$, $u = 1$. So

$$A_1 = \int_0^1 e^{\sqrt{x}}dx = \int_0^1 e^u(2udu) = 2\int_0^1 ue^u du$$

b.

$$A_2 = \int_0^1 2xe^x dx = 2\int_0^1 ue^u du$$

c. Let $u = \sin x$, so $du = \cos x dx$. When $x = 0$, $u = 0$, and when $x = \frac{\pi}{2}$, $u = 1$. So

$$A_3 = \int_0^{\pi/2} e^{\sin x} dx \sin 2x dx = \int_0^{\pi/2} e^{\sin x}(2\sin x \cos x)dx = \int_0^1 e^u(2udu) = 2\int_0^1 ue^u du$$

Since $A_1 = A_2 = A_3$, all three areas are equal.

65. Let $u = 1 - x$. Then $x = 1 - u$ and $dx = -du$, so

$$\int_0^1 x^a(1-x)^b dx = \int_1^0 (1-u)^a u^b(-du) = \int_0^1 u^b(1-u)^a du = \int_0^1 x^b(1-x)^a dx$$