

## Mathematics 1a, Section 5.3 Solutions

Alexander Ellis

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2.  $\int_a^b I(t)dt = \int_a^b Q'(t)dt = Q(b) - Q(a)$  by the Total Change Theorem, so it represents the change in the charge  $Q$  from time  $t = a$  to  $t = b$ .

8. The units for  $a(x)$  are pounds per foot and the units for  $x$  are feet, so the units for  $da/dx$  are pounds per foot per foot, denoted (lb/ft)/ft. The unit of measurement for  $\int_2^8 a(x)dx$  is the product of pounds per foot and feet; that is, pounds.

12.

$$\int_0^4 (1 + 3y - y^2)dy = \left[ y + \frac{3}{2}y^2 - \frac{1}{3}y^3 \right]_0^4 = \left( 4 + \frac{3}{2} \cdot 16 - \frac{1}{3} \cdot 64 \right) - (0) = \frac{20}{3}$$

16.

$$\int_0^1 x^{3/7} dx = \left[ \frac{x^{10/7}}{10/7} \right]_0^1 = \left[ \frac{7}{10} x^{10/7} \right]_0^1 = \frac{7}{10} - 0 = \frac{7}{10}$$

26.

$$\begin{aligned} \int_1^8 \frac{x-1}{\sqrt[3]{x^2}} dx &= \int_1^8 (x^{1/3} - x^{-2/3}) dx \\ &= \left[ \frac{x^{4/3}}{4/3} - \frac{x^{1/3}}{1/3} \right]_1^8 = \left[ \frac{3}{4} x^{4/3} - 3x^{1/3} \right]_1^8 \\ &= \left( \frac{3}{4} \cdot 16 - 3 \cdot 2 \right) - \left( \frac{3}{4} - 3 \right) = \frac{33}{4} \end{aligned}$$

48.

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx = \int 2 \cos x dx = 2 \sin x + C$$

**56.**

$$n(10) - n(4) = \int_4^{10} (200 + 50t)dt = [200t + 25t^2]_4^{10} = 2000 + 2500 - (800 + 400) = 3300$$

**64.**  $B = 3A$ , so:

$$\int_0^b e^x dx = 3 \int_0^a e^x dx$$

$$[e^x]_0^b = 3[e^x]_0^a$$

$$e^b - 1 = 3(e^a - 1)$$

$$e^b = 3e^a - 2$$

$$b = \ln(3e^a - 2)$$