Mathematics 1a, Section 5.2 Solutions

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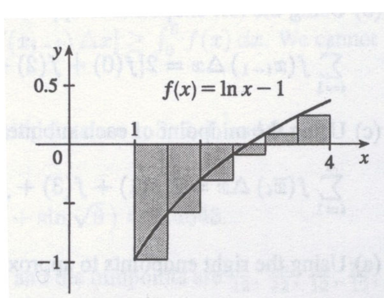
2. For this problem, $x_i^* = x_{i-1}$ is a left endpoint and $\Delta x = 0.5$.

$$
\begin{aligned}
L_6 &= \sum_{i=1}^{6} f(x_{i-1})\Delta x \\
&= 0.5\left[f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5)\right] \\
&\approx 0.5(-1 - 0.5945349 - 0.3068528 - 0.0837093 + 0.0986123 + 0.2527630) \\
&= 0.5(-1.6337217) \approx -0.816861
\end{aligned}
$$

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the four rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.



6. a. Using the right endpoints to approximate $\int_{-3}^{3} g(x)dx$, we have

$$
\begin{aligned}
\sum_{i=1}^{6} g(x_i)\Delta x &= 1\left[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)\right] \\
&\approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5
\end{aligned}
$$

b. Using the left endpoints to approximate $\int_{-3}^{3} g(x)dx$, we have

$$\sum_{i=1}^{6} g(x_{i-1})\Delta x = 1\,[g(-3)+g(-2)+g(-1)+g(0)+g(1)+g(2)]$$
$$\approx 2+1-0.5-1.5-1.5-0.5=-1$$

c. Using the midpoint of each subinterval to approximate $\int_{-3}^{3} g(x)dx$, we have

$$\sum_{i=1}^{6} g(\bar{x}_i)\Delta x = 1\,[g(-2.5)+g(-1.5)+g(-0.5)+g(0.5)+g(1.5)+g(2.5)]$$
$$\approx 1.5+0-1-1.75-1+0.5=-1.75$$

10. $\Delta x = (\pi - 0)/6 = \frac{\pi}{6}$, so the endpoints are $0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$ and the midpoints are $\frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}$. The Midpoint Rule gives:

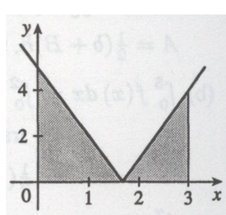
$$\int_0^{\pi} \sec(x/3)dx \approx \sum_{i=1}^{6} f(\bar{x}_i)\Delta x = \frac{\pi}{6}\left(\sec\frac{\pi}{36}+\sec\frac{3\pi}{36}+\sec\frac{5\pi}{36}+\sec\frac{7\pi}{36}+\sec\frac{9\pi}{36}+\sec\frac{11\pi}{36}\right) \approx 3.9379$$

16. $\int_0^2 e^{-x^2}dx$ with $n = 5, 10, 50, 100$.

n	L_n	R_n
5	1.077467	0.684794
10	0.980007	0.783670
50	0.901705	0.862438
100	0.891896	0.872262

The value of the integral lies between 0.872 and 0.892. Note that $f(x) = e^{-x^2}$ is decreasing on $(0, 2)$. We cannot make a similar statement for $\int_{-1}^{2} e^{-x^2}dx$ since f is increasing on $(-1, 0)$.

36. $\int_0^3 |3x-5|dx$ can be interpreted as the area under the graph of the function $f(x) = |3x-5|$ between $x = 0$ and $x = 3$. This is equal to the sum of the areas of the two triangles, so $\int_0^3 |3x-5|dx = \frac{1}{2}\cdot\frac{5}{3}\cdot 5 + \frac{1}{2}\left(3-\frac{5}{3}\right)4 = \frac{41}{6}$.



40.

$$\int_2^{10} f(x)dx - \int_2^7 f(x)dx = \left(\int_2^7 f(x)dx + \int_7^{10} f(x)dx\right) - \int_2^7 f(x)dx = \int_7^{10} f(x)dx$$

46. Using Integral Comparison Property 8,

$$m \le f(x) \le M$$
$$m(2-0) \le \int_0^2 f(x)dx \le M(2-0)$$
$$2m \le \int_0^2 f(x)dx \le 2M$$

47. On $[1, 3]$, $\ln 1 \le \ln x \le \ln 3$, so

$$0(3-1) \le \int_1^3 \ln x dx \le \ln 3(3-1)$$
$$0 \le \int_1^3 \ln x dx \le 2\ln 3$$

49.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^4} \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^4 \frac{1}{n}$$

At this point, we need to recognize the limit as being of the form $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, where $\Delta x = (1-0)/n = 1/n$, $x_i = 0 + i\Delta x = i/n$, and $f(x) = x^4$. Thus, the definite integral is $\int_0^1 x^4 dx$.