Mathematics 1a, Section 5.1 Solutions

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2. a. i. In this case, $\Delta x = \frac{12-0}{6} = 2.$

$$
\begin{aligned}
L_6 &= \sum_{i=1}^{6} f(x_{i-1})\Delta x \\
&= 2\left[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right] \\
&= 2\left[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)\right] \\
&\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) \\
&= 2(43.3) = 86.6
\end{aligned}
$$

ii.

$$
\begin{aligned}
R_6 &= L_6 + 2 \cdot f(12) - 2 \cdot f(0) \\
&= 86.6 + 2(1) - 2(9) = 70.6
\end{aligned}
$$

iii.

$$
\begin{aligned}
M_6 &= \sum_{i=1}^{6} f(x_i^*)\Delta x \\
&= 2\left[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)\right] \\
&\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \\
&= 2(39.7) = 79.4
\end{aligned}
$$

b. Since f is decreasing, we obtain an overestimate by using left endpoints; that is, L_6.

c. Since f is decreasing, we obtain an underestimate by using right endpoints; that is, R_6.

d. M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimate and underestimate in L_6 and R_6.

4. a. In this case, $\Delta x = \frac{5-0}{5} = 1$.

$$
\begin{aligned}
R_5 &= \sum_{i=1}^{5} f(x_i)\Delta x \\
&= f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1 + f(x_4)\cdot 1 + f(x_5)\cdot 1 \\
&= f(1) + f(2) + f(3) + f(4) + f(5) \\
&= 24 + 21 + 16 + 9 + 0 = 70
\end{aligned}
$$

Since f is decreasing on $[0,5]$, R_5 is an underestimate.

b.

$$
\begin{aligned}
L_5 &= \sum_{i=1}^{5} f(x_{i-1}\Delta x \\
&= f(0) + f(1) + f(2) + f(3) + f(4) \\
&= 25 + 24 + 21 + 16 + 9 = 95
\end{aligned}
$$

L_5 is an overestimate.

6. a.

b. $f(x) = e^{-x^2}$ and $\Delta x = \frac{2-(-2)}{4} = 1$.

$$
\begin{aligned}
R_4 &= 1\cdot f(-1) + 1\cdot f(0) + 1\cdot f(1) + 1\cdot f(2) \\
&= e^{-1} + 1 + e^{-1} + e^{-4} \\
&\approx 1.754 \\
M_4 &= 1\cdot f(-1.5) + 1\cdot f(-0.5) + 1\cdot f(0.5) + 1\cdot f(1.5) \\
&= e^{-2.25} + e^{-0.25} + e^{-0.25} + e^{-2.25} \\
&\approx 1.768
\end{aligned}
$$

c.

$$
\begin{aligned}
R_8 &= 0.5[f(-1.5) + f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\
&= e^{-2.25} + e^{-1} + e^{-0.25} + 1 + e^{-0.25} + e^{-1} + e^{-2.25} + e^{-4} \\
&\approx 1.761
\end{aligned}
$$

For the midpoints, we see that the figure is symmetric about the y-axis, so

$$\begin{aligned} M_8 &= (0.5)(2)[f(0.25) + f(0.75) + f(1.25) + f(1.75)] \\ &= e^{-0.0625} + e^{-0.5625} + e^{-1.5625} + e^{3.0625} \\ &\approx 1.766 \end{aligned}$$

7. This problem wasn't assigned, but its solution is used in **8**, so portion of it is presented.

First let SUM = 0, X_MIN = 0, X_MAX = π, N equal whatever number of terms you want in the sum, DELTA_X = (X_MAX − X_MIN)/N, and RIGHT_ENDPOINT = X_MIN + DELTA_X.

Repeat steps "a" and "b" until RIGHT_ENDPOINT > X_MAX:

Step "a": Add sin(RIGHT_ENDPOINT) to SUM.

Step "b": Add DELTA_X to RIGHT_ENDPOINT.

At the end of this procedure, the quantity (DELTA_X)(SUM) is the answer we are looking for.

8. We can use the algorithm from **7** with X_MIN = 1, X_MAX = 2, and 1/(RIGHT_ENDPOINT)^2 instead of sin(RIGHT_ENDPOINT) in "a". We find that

$$R_{10} = \frac{1}{10}\sum_{i=1}^{10}\frac{1}{(1+i/10)^2} \approx 0.4640$$

$$R_{30} = \frac{1}{30}\sum_{i=1}^{30}\frac{1}{(1+i/30)^2} \approx 0.4877$$

$$R_{50} = \frac{1}{50}\sum_{i=1}^{50}\frac{1}{(1+i/50)^2} \approx 0.4926$$

It appears that the exact area is $\frac{1}{2}$.

12. We can find an upper estimate by using the final velocity for each time interval. Thus, the distance d traveled after 62 seconds can be approximated by

$$d = \sum_{i=1}^{6} v(t_i)\Delta t_i = (185\text{ft/s})(10\text{s}) + 319\cdot 5 + 447\cdot 5 + 742\cdot 12 + 1325\cdot 27 + 1445\cdot 3 = 54,694\text{ft}$$

18. a. $\Delta x = \frac{1-0}{n} = \frac{1}{n}$ and $x_i = 0 + i\Delta x = \frac{i}{n}$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

b.

$$\begin{aligned} \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3}\frac{1}{n} &= \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3 \\ &= \lim_{n\to\infty} \frac{1}{n^4} \left[\frac{n(n+1)}{2}\right]^2 \\ &= \lim_{n\to\infty} \frac{(n+1)^2}{4n^2} = \frac{1}{4} \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^2 = \frac{1}{4} \end{aligned}$$