Mathematics 1a, Section 3.8 Solutions

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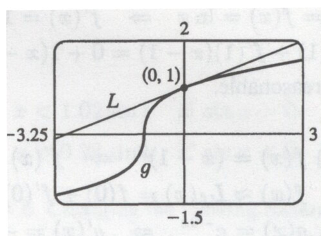
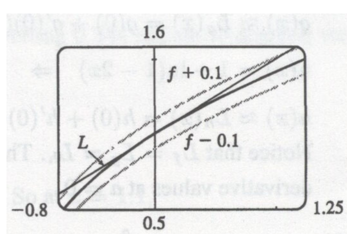
1. $f(x) = x^3$, so $f'(x) = 3x^2$, so $f(1) = 1$ and $f'(1) = 3$. With $a = 1$, $L(x) = f(a) + f'(a)(x - a)$ becomes

$$L(x) = 1 + 3(x - 1) = 3x - 2$$

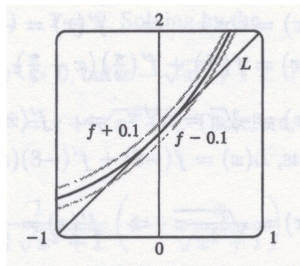
6. $g(x) = \sqrt[3]{1+x} = (1+x)^{1/3}$, so $g'(x) = \frac{1}{3}(1+x)^{-2/3}$, so $g(0) = 1$ and $g'(0) = \frac{1}{3}$. Therefore, $\sqrt[3]{1+x} = g(x) \approx g(0) + g'(0)(x - 0) = 1 + \frac{1}{3}x$. So $\sqrt[3]{0.95} = \sqrt[3]{1 + (-0.05)} \approx 1 + \frac{1}{3}(-0.05) = 0.98\bar{3}$, and $\sqrt[3]{1.1} = \sqrt[3]{1 + 0.1} \approx 1 + \frac{1}{3}(0.1) = 1.0\bar{3}$.



7. $f(x) = \sqrt{1+x}$, so $f'(x) = \frac{1}{2\sqrt{1+x}}$, so $f(0) = 1$ and $f'(0) = \frac{1}{2}$. Thus, $f(x) \approx f(0) + f'(0)(x - 0) = 1 + \frac{1}{2}x$. We need $\sqrt{1+x} - 0.1 < 1 + \frac{1}{2}x < \sqrt{1+x} + 0.1$. By zooming in or using an intersect feature, we see that this is true when $-0.69 < x < 1.09$.



10. $f(x) = e^x$, so $f'(x) = e^x$, thus $f(0) = f'(0) = 1$. Thus $f(x) \approx f(0) + f'(0)(x - 0) = 1 + x$. We need $e^x - 0.1 < 1 + x < e^x + 0.1$, which is true when $-0.483 < x < 0.416$.



14. a. $f(x) = (x-1)^2$, so $f'(x) = 2(x-1)$, so $f(0) = 1$ and $f'(0) = -2$. Thus

$$f(x) \approx L_f(x) = f(0) + f'(0)(x-0) = 1 - 2x$$

$g(x) = e^{-2x}$, so $g'(x) = -2e^{-2x}$, so $g(0) = 1$ and $g'(0) = -2$. Thus

$$g(x) \approx L_g(x) = g(0) + g'(0)(x-0) = 1 - 2x$$

$h(x) = 1 + \ln(1-2x)$, so $h'(x) = -\frac{2}{1-2x}$, so $h(0) = 1$ and $h'(0) = -2$. Thus

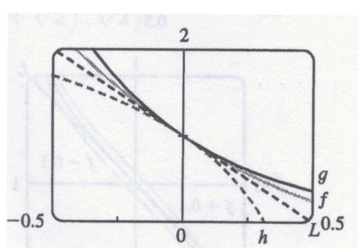
$$h(x) \approx L_h(x) = h(0) + h'(0)(x-0) = 1 - 2x$$

Note that $L_f = L_g = L_h$. This happens because f, g, h have the same function values and the same derivative values at $a = 0$.

b. The linear approximation appears to be the best for the function f because it is closer to f for a larger domain than it is to g and h. The approximation looks worst for h since h moves away from L faster than f and g do.

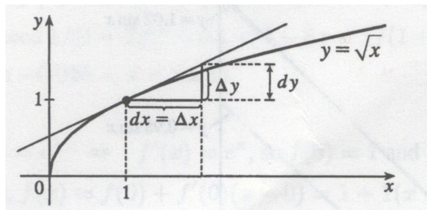
18. a. $y = \sqrt{x}$, so $dy = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$.

b.



$x = 1$ and $dx = 1$ implies $dy = \frac{1}{2(1)}(1) = \frac{1}{2}$. $\Delta y = f(x + \Delta x) - f(x) = \sqrt{1+1} - \sqrt{1} = \sqrt{2} - 1 \approx 0.414$.

c.



Remember, Δy represents the amount that the curve $y = f(x)$ rises or falls when x changes by an amount dx, where as dy represents the amount that the tangent line rises or falls (the change in the linearization).

20. **a.** $A = \pi r^2$, so $dA = 2\pi r dr$. When $r = 24$ and $dr = 0.2$, $dA = 2\pi(24)(0.2) = 9.6\pi$, so the maximum possible error in the calculated area of the disk is about $9.6\pi \approx 30\text{cm}^2$.

b. Relative error is

$$\frac{\Delta A}{A} \approx \frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = \frac{2dr}{r} = \frac{2(0.2)}{24} = \frac{1}{60} = 0.01\bar{6}$$

Percentage error is relative error times 100%, or $1.\bar{6}\%$.

22. $F = kR^4$, so $dF = 4kR^3 dR$, thus

$$\frac{dF}{F} = \frac{4kR^3 dR}{kR^4} = 4\frac{dR}{R}$$

Thus, the relative change in F is about 4 times the relative change in R. So a 5% increase in the radius corresponds to a 20% increase in blood flow.