

Mathematics 1a, Section 3.7 Solutions

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November 4, 2004

4.

$$f(x) = \cos(\ln x)$$
$$f'(x) = -\sin(\ln x) \frac{1}{x} = -\frac{\sin(\ln x)}{x}$$

6.

$$f(x) = \log_{10} \left(\frac{x}{x-1} \right) = \log_{10} x - \log_{10}(x-1)$$
$$f'(x) = \frac{1}{x \ln 10} - \frac{1}{(x-1) \ln 10} = -\frac{1}{x(x-1) \ln 10}$$

12.

$$h(x) = \ln \left(x + \sqrt{x^2 - 1} \right)$$
$$h'(x) = \frac{1}{x + \sqrt{x^2 - 1}} \left(1 + \frac{x}{\sqrt{x^2 - 1}} \right) = \frac{1}{x + \sqrt{x^2 - 1}} \cdot \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}} = \frac{1}{\sqrt{x^2 - 1}}$$

18.

$$y = [\ln(1 + e^x)]^2$$
$$y' = 2 \ln(1 + e^x) \frac{1}{1 + e^x} e^x = \frac{2e^x \ln(1 + e^x)}{1 + e^x}$$

20.

$$y = \ln(\sec x + \tan x)$$
$$y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x$$
$$y'' = \sec x \tan x$$

28.

$$\begin{aligned}y &= \sqrt{x}e^{x^2}(x^2 + 1)^{10} \\ \ln y &= \ln \sqrt{x} + \ln e^{x^2} + \ln(x^2 + 1)^{10} \\ \ln y &= \frac{1}{2} \ln x + x^2 + 10 \ln(x^2 + 1) \\ \frac{1}{y} \ln y &= \frac{1}{2x} + 2x + 10 \frac{1}{x^2 + 1} (2x) \\ y' &= \sqrt{x}e^{x^2}(x^2 + 1)^{10} \left(\frac{1}{2x} + 2x + \frac{20x}{x^2 + 1} \right)\end{aligned}$$

32.

$$\begin{aligned}y &= x^{1/x} \\ \ln y &= \frac{1}{x} \ln x \\ \frac{y'}{y} &= \frac{1}{x} \left(\frac{1}{x} \right) + (\ln x) \left(-\frac{1}{x^2} \right) \\ y' &= x^{1/x} \frac{1 - \ln x}{x^2}\end{aligned}$$

34.

$$\begin{aligned}y &= (\sin x)^x \\ \ln y &= x \ln(\sin x) \\ \frac{y'}{y} &= x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 1 \\ y' &= (\sin x)^x [x \cot x + \ln(\sin x)]\end{aligned}$$