Mathematics 1a, Section 3.5 Solutions

Alexander Ellis

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2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u}$. Then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$$

4. Let $u = g(x) = \sin x$ and $y = f(u) = \tan u$. Then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \left(\sec^2 u\right)(\cos x) = \sec^2(\sin x)\cos x$$

14.

$$y = 4\sec 5x$$
$$y' = 4\sec 5x \tan 5x(5) = 20\sec 5x\tan 5x$$

26.

$$\begin{aligned} y &= \tan^2(3\theta) = (\tan 3\theta)^2 \\ y' &= 2(\tan 3\theta)\cdot\frac{d}{d\theta}(\tan 3\theta) \\ &= 2\tan 3\theta \cdot \sec^2 3\theta \cdot 3 \\ &= 6\tan 3\theta \sec^2 3\theta \end{aligned}$$

32.

$$y = x^2 e^{-x}$$
$$y' = x^2\left(-e^{-x}\right) + e^{-x}(2x) = 2xe^{-x} - x^2e^{-x}$$

At $(1, \frac{1}{e})$, $y' = 2e^{-1} - e^{-1} = \frac{1}{e}$. So an equation of the tangent line is $y = \frac{1}{e}x$.

42. **a.** $h(x) = f(f(x))$, thus $h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

b. $g(x) = f(x^2)$, thus $g'(x) = f'(x^2) \cdot 2x$, so $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(1.5) = 6$.

52.

$$\begin{aligned} y &= e^{rx} \\ y' &= re^{rx} \\ y'' &= r^2e^{rx} \end{aligned}$$

so

$$\begin{aligned} y'' + 5y' - 6y &= r^2e^{rx} + 5re^{rx} - 6e^{rx} \\ &= e^{rx}(r^2 + 5r - 6) \\ &= e^{rx}(r+6)(r-1) = 0 \\ &\Rightarrow (r+6)(r-1) = 0 \\ &\Rightarrow r = \{1, -6\} \end{aligned}$$

54.

$$\begin{aligned} f(x) &= xe^{-x} \\ f'(x) &= e^{-x} - xe^{-x} \\ f''(x) &= -e^{-x} + (1-x)(-e^{-x}) = (x-2)e^{-x} \\ f'''(x) &= \ldots = (3-x)e^{-x} \\ f^{(4)}(x) &= (4-x)e^{-x} \\ &\ldots \\ f^{(1000)}(x) &= (x-1000)e^{-x} \end{aligned}$$

56. **a.** $s = A\cos(\omega t + \delta)$, thus velocity is $v = s' = -\omega A \sin(\omega t + \delta)$.

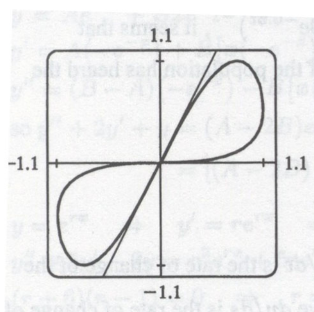
b. If $A \neq 0$ and $\omega \neq 0$, then $s' = 0$ if and only if $\sin(\omega t + \delta) = 0$, that is, $\omega t + \delta = n\pi$, where n is any integer. So

$$t = \frac{n\pi - \delta}{\omega}$$

66. $x = \sin t$ and $y = \sin(t + \sin t)$. We work at the origin, $(x, y) = (0, 0)$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy/dt}{dx/dt} \\
&= \frac{\cos(t + \sin t)(1 + \cos t)}{\cos t} \\
&= \cos(t + \sin t)\frac{1 + \cos t}{\cos t} \\
&= \cos(t + \sin t)\left(\frac{1}{\cos t} + 1\right) \\
&= (\sec t + 1)\cos(t + \sin t)
\end{aligned}$$

Now $x = \sin t$ is 0 when $t = 0$ and $t = \pi$, so there are two tangents at the point $(0, 0)$ since both $t = 0$ and $t = \pi$ correspond to the origin. The tangent corresponding to $t = 0$ has slope $(\sec 0 + 1)\cos(0 + \sin 0) = 2\cos 0 = 2$, and its equation is $y = 2x$. The tangent corresponding to $t = \pi$ has slope $(\sec \pi + 1)\cos(\pi + \sin \pi) = 0$, so it is the x-axis, that is, $y = 0$.



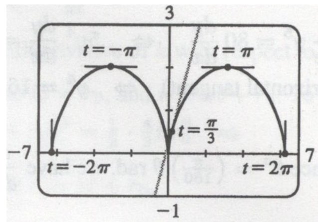
68. a. $x = r(\theta - \sin\theta)$ and $y = r(1 - \cos\theta)$, so

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r(\sin\theta)}{r(1 - \cos\theta)} = \frac{\sin\theta}{1 - \cos\theta}$$

When $\theta = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{\sqrt{3}/2}{1-1/2} = \sqrt{3}$ and $(x, y) = \left(r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right), r\frac{1}{2}\right)$, and the tangent is

$$y - \frac{1}{2}r = \sqrt{3}\left[x - r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right]$$

b. Horizontal tangents occur when $\frac{dy}{dx} = 0$, which means $\sin\theta = 0$ and $\cos\theta \neq 1$, which means $\theta = (2n+1)\pi$. The corresponding points are $((2n+1)\pi r, 2r)$. A vertical tangent means $\frac{dy}{dx}$ is undefined, which means $1 - \cos\theta = 0$, so $\cos\theta = 1$, thus $\theta = 2n\pi$. The corresponding points are $(2n\pi r, 0)$.


3
t = −π
t = π
t = π/3
−7
7
t = −2π
t = 2π
−1