Mathematics 1a, Section 3.4 Solutions

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6.

$$y = e^u(\cos u + cu)$$
$$y' = e^u(-\sin u + c) + (\cos u + cu)e^u = e^u(\cos u - \sin u + cu + c)$$

18.

$$y = e^x \cos x$$
$$y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x)$$

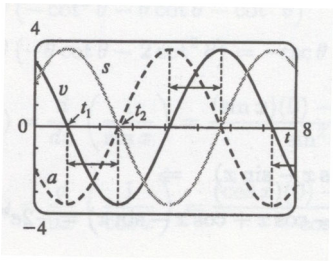
So the slope of the tangent line at $(0, 1)$ is $e^0(\cos 0 - \sin 0) = 1(1 - 0) = 1$ and an equation is

$$y - 1 = 1(x - 0)$$
$$y = x + 1$$

30. a.

$$s(t) = 2\cos t + 3\sin t$$
$$v(t) = -2\sin t + 3\cos t$$
$$a(t) = -2\cos t - 3\sin t$$

b.



c. $s = 0$, so $t_2 \approx 2.55$. So the mass passes through the equilibrium position for the first time when $t \approx 2.55$s.

d. $v = 0$, so $t_1 \approx 0.98$, $s(t_1) \approx 3.61$cm. So the mass travels at a maximum of about 3.6cm (upward and downward) from its equilibrium position.

e. The speed $|v|$ is greatest when $s = 0$; that is, when $t = t_2 + n\pi$, n a positive integer. The mass is speeding up when v and a have the same sign. From the figure, we see that this is the case on the intervals $(t_1 + n\pi, t_2 + n\pi)$ where n is a whole number.

38.

$$\begin{aligned}
\lim_{x\to 0} x\cot x &= \lim_{x\to 0} x \cdot \frac{\cos x}{\sin sx} \\
&= \lim_{x\to 0} \frac{x\cos x}{\sin x} \\
&- \lim_{x\to 0} \frac{\frac{1}{x}x\cos x}{\frac{1}{x}\sin x} \\
&= \lim_{x\to 0} \frac{\cos x}{\frac{1}{x}\sin x} \\
&= \frac{\lim_{x\to 0}\cos x}{\lim_{x\to 0}\frac{1}{x}\sin x} \\
&= \frac{1}{1} = 1
\end{aligned}$$