Mathematics 1a, Section 3.2 Solutions

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1. Product rule:

$$y = (x^2+1)(x^3+1)$$
$$y' = (x^2+1)(3x^2) + (x^3+1)(2x) = 3x^4 + 3x^2 + 2x^4 + 2x = 5x^4 + 3x^2 + 2x$$

Multiplying first:

$$y = (x^2+1)(x^3+1) = x^5 + x^3 + x^2 + 1$$
$$y' = 5x^4 + 3x^2 + 2x$$

The two are equivalent.

2. Quotient rule:

$$F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}} = \frac{x - 3x^{3/2}}{x^{1/2}}$$
$$F'(x) = \frac{x^{1/2}(1 - \frac{9}{2}x^{1/2}) - (x - 3x^{3/2})(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}$$
$$= \frac{\frac{1}{2}x^{1/2} - 3x}{x} = \frac{1}{2}x^{-1/2} - 3$$

Simplifying first:

$$F(x) = \frac{x - 3x^{3/2}}{x^{1/2}}$$
$$F'(x) = \frac{1}{2}x^{-1/2} - 3$$

They are equivalent. Clearly, simplifying first is easier.

6. By the Quotient Rule:

$$y = \frac{e^x}{1+x}$$

$$y' = \frac{(1+x)e^x - e^x(1)}{(1+x)^2} = \frac{e^x + xe^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

18.

$$f(x) = \frac{ax+b}{cx+d}$$

$$f'(x) = \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} = \frac{acx + ad - acx - bc}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$$

31. a. From the graphs of f and g, we obtain the following values: $f(1) = 2$ since the point $(1, 2)$ is on the graph of f; $g(1) = 1$ since the point $(1, 1)$ is on the graph of g; $f'(1) = 2$ since the slope of the line segment between $(0, 0)$ and $(2, 4)$ is 2; $g'(1) = -1$ since the slope of the line segment between $(-2, 4)$ and $(2, 0)$ is -1. Now $u(x) = f(x)g(x)$, so $u'(1) = f(1)g'(1) + f'(1)g(1) = 0$.

b.

$$v(x) = \frac{f(x)}{g(x)}$$

$$v'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{2(-\frac{1}{3}) - 3(\frac{2}{3})}{2^2} = \frac{2}{3}$$

34. a. $f(20) = 10,000$ means that when the price of fabric is \$200 per yard, 10,000 yards will be sold. $f'(20) = -350$ means that as the price of the fabric increases past \$20 per yard, the amount of fabric which will be sold is decreasing at a rate of 350 yards per (dollar per yard).

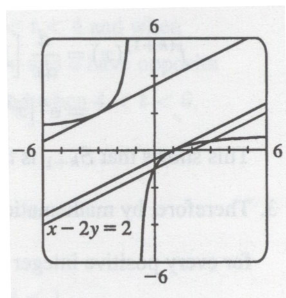
b. $R(p) = pf(p)$, so $R'(p) = pf'(p) + f(p)$. Thus $R'(20) = 20f'(20) + f(20) = 20(-350) + 10,000 = 3,000$. This means that as the price of fabric increases past \$20 per yard, the total revenue is increasing at \$3000 per dollards per yard. Note that the product rule indicates that we will lose \$7000 per dollar per yard due to selling less fabric, but that that loss is more than made up for by the additional revenue due to the increase in price.

38.

$$y = \frac{x-1}{x+1}$$

$$y' = \frac{(x+1)(1) + (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

If the tangent intersects the curve when $x = a$, then its slope is $\frac{2}{(a+1)^2}$. But if the tangent is parallel to $x - 2y = 2$, that is, $y = \frac{1}{2}x - 1$, then its slope is $\frac{1}{2}$. Thus, $\frac{2}{(a+1)^2} = \frac{1}{2}$, so $(a+1)^2 = 4$, so $a+1 = \pm 2$, so $a = 1$ or $a = -3$. When $a = 1$, $y = 0$ and the equation of the tangent is $y = \frac{1}{2}x - \frac{1}{2}$. When $a = -3$, $y = 2$ and the equation of the tangent is $y = \frac{1}{2}x + \frac{7}{2}$.



42. a. Let $f(x) = 1/g(x)$. By the definition of the derivative:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{1/g(x+h) - 1/g(x)}{h} \\ &= \lim_{h\to 0} \frac{g(x) - g(x+h)}{hg(x)g(x+h)} \\ &= -\lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \lim_{h\to 0} \frac{1}{g(x)g(x+h)} \\ &= -\frac{g'(x)}{[g(x)]^2} \end{aligned}$$

Thus,

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$$

b.

$$\begin{aligned} y &= \frac{1}{s + ke^s} \\ y' &= -\frac{1 + ke^s}{(s + ke^s)^2} \end{aligned}$$