Mathematics 1a, Section 2.5 Solutions

Alexander Ellis

October 14, 2004

1. a. As x approaches 2 (from the right or the left), the values of $f(x)$ become large.
b. As x approaches 1 from the right, the values of $f(x)$ become large and negative.
c. As x becomes large, the values of $f(x)$ approach 5.
d. As x becomes large and negative, the values of $f(x)$ approach 3.

4. a.

$$\lim_{x\to\infty} g(x) = 2$$

b.

$$\lim_{x\to-\infty} g(x) = -2$$

c.

$$\lim_{x\to 3} g(x) = \infty$$

d.

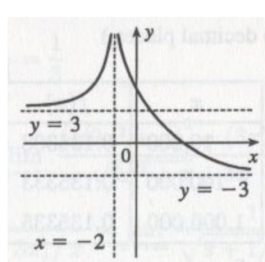
$$\lim_{x\to 0} g(x) = -\infty$$

e.

$$\lim_{x\to -2^+} g(x) = -\infty$$

f. Vertical: $x = -2, , x = 0, x = 3$. Horizontal: $y = -2, y = 2$.

8.



$$\lim_{x\to -2} f(x) = \infty$$

$$\lim_{x\to \infty} f(x) = -3$$

$$\lim_{x\to -\infty} f(x) = -3$$

10. a.

x	$f(x)$
0.5	-1.14
0.9	-3.69
0.99	-33.7
0.999	-333.7
0.9999	-3333.7
0.99999	-33,333.7

x	$f(x)$
1.5	0.42
1.1	3.02
1.01	33.0
1.001	333.0
1.0001	3333.0
1.00001	33,333.3

From these calculations, it seems that $\lim_{x\to 1^-} f(x) = -\infty$ and $\lim_{x\to 1^+} f(x) = \infty$.

b. If x is slightly smaller than 1, then $x^3 - 1$ will be a negative number close to 0, and the reciprocal of $x^3 - 1$, that is, $f(x)$, will be a negative number with large absolute value. So $\lim_{x\to x^-} f(x) = -\infty$.

If x is slightly larger than 1, then $x^3 - 1$ will be a small positive number, and its reciprocal, $f(x)$, will be a large positive number. So $\lim_{x\to 1^+} f(x) = \infty$.

c. It appears from the graph of f that

$$\lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to 1^+} f(x) = \infty$$

12. a. From a graph of $f(x) = (1 - 2/x)^x$ in a window of [0, 10,000] by [0,0.2], we estimate that $\lim_{x\to\infty} f(x) = 0.14$, to two decimal places.

b.

x	$f(x)$
10,000	0.135308
100,000	0.135333
1,000,000	0.135335

From the table, we estimate that $\lim_{x\to\infty} f(x) = 0.1353$ to four decimal places.

14.

$$\lim_{x\to 5^-} \frac{e^x}{(x-5)^3} = -\infty$$

since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 5^-$.

22.

$$\lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}}=\lim_{x\to\infty}\frac{(x+2)/2}{\sqrt{9x^2+1}/\sqrt{x^2}}=\lim_{x\to\infty}\frac{1+2/x}{\sqrt{9+1/x^2}}=\frac{1+0}{\sqrt{9+0}}=\frac{1}{3}$$

23.

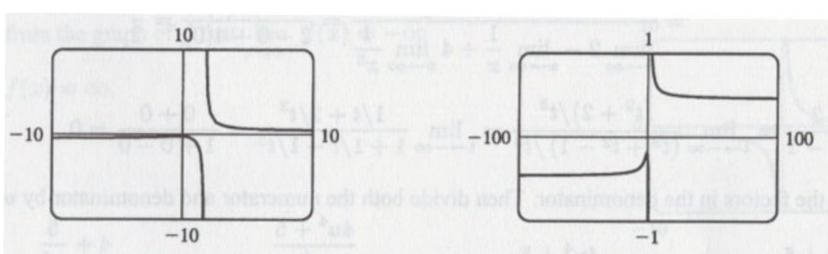
$$\begin{aligned}\lim_{x\to\infty}(\sqrt{9x^2+x}-3x)&=\lim_{x\to\infty}\frac{(\sqrt{9x^2+x}-3x)(\sqrt{9x^2+x}+3x)}{(\sqrt{9x^2+x}+3x)}\\&=\lim_{x\to\infty}\frac{(\sqrt{9x^2+x})^2-(3x)^2}{\sqrt{9x^2+x}+3x}\\&=\lim_{x\to\infty}\frac{x/x}{(\sqrt{9x^2+x}+3x)/x}\\&=\lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3}\\&=\frac{1}{\sqrt{9}+3}=\frac{1}{6}\end{aligned}$$

27.

$$\lim_{x\to\infty}\frac{x^7-1}{x^6+1}=\lim_{x\to\infty}\frac{1-1/x^7}{1/x+1/x^7}=\infty$$

since $1-1/x^7\to 1$ while $1/x+1/x^7\to 0^+$ as $x\to\infty$. Or divide numerator and denominator by x^6 instead of x^7.

30. a.



From the graph, it appears at first there is only one horizontal asymptote, as $y\approx 0$, and a vertical asymptote as $x\approx 1.7$. However, if we graph the function with a wider viewing rectangle, we see that in fact there seem to be two horizontal asymptotes: one at $y\approx 0.5$ and one at $y\approx -0.5$. So we estimate that

$$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}\approx 0.5$$

$$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}\approx -0.5$$

b. $f(1000) \approx 0.4722$ and $f(10,000) \approx 0.4715$, so we estimate that $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5} \approx 0.47$. $f(-1000) \approx -0.4706$ and $f(-10,000) \approx -0.4713$, so we estimate that $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5} \approx -0.47$.

c.

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5} = \lim_{x\to\infty} \frac{\sqrt{2+1/x^2}}{3-5/x} = \frac{\sqrt{2}}{3} \approx 0.471404$$

since $\sqrt{x^2} = x$ for $x > 0$. For $x < 0$ we have $\sqrt{x^2} = |x| = -x$, so when we divide the numerator by x, with $x < 0$, we get

$$\frac{1}{x}\sqrt{x^2+1} = -\frac{1}{\sqrt{x^2}}\sqrt{2x^2+1} = -\sqrt{2+1/x^2}$$

Therefore,

$$\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{3x-5} = \lim_{x\to-\infty} \frac{-\sqrt{2+1/x^2}}{3-5/x} = -\frac{\sqrt{2}}{3} \approx -0.471404$$

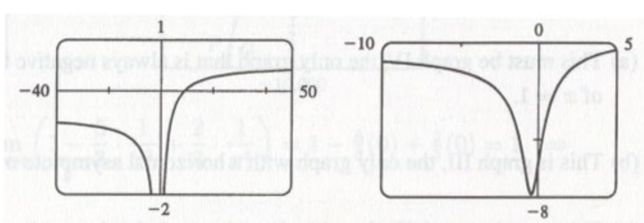
32.

$$\lim_{x\to\infty} \frac{x-9}{\sqrt{4x^2+3x+2}} = \lim_{x\to\infty} \frac{1-9/x}{\sqrt{4+3/x+2/x^2}} = \frac{1-0}{\sqrt{4+0+0}} = \frac{1}{2}$$

Using the fact that for $x < 0$ we have $\sqrt{x^2} = |x| = -x$, we divide the numerator by $-x$ and the denominator by $\sqrt{x^2}$. Thus

$$\lim_{x\to-\infty} \frac{x-9}{\sqrt{4x^2+3x+2}} = \lim_{x\to-\infty} \frac{-1+9/x}{\sqrt{4+3/x+2/x^2}} = \frac{-1+0}{\sqrt{4+0+0}} = -\frac{1}{2}$$

The horizontal asymptotes are $y = \pm\frac{1}{2}$. The polynomial $4x^2+3x+2$ is positive for all x, so the denominator never approaches zero, and thus there is no vertical asymptote.



36. Since the function has vertical asymptotes $x = 1$ and $x = 3$, the denominator of the rational function we are looking for must have $(x-1)$ and $(x-3)$. Because the horizontal asymptote is $y = 1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the lading coefficients must be 1. One possibility is

$$f(x) = \frac{x^2}{(x-1)(x-3)}$$