Mathematics 1a, Section 2.3 Solutions

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October 4, 2004

2. a.

$$\lim_{x\to 2}(f(x)+g(x)) = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2+0 = 2$$

b. $\lim_{x\to 1} g(x)$ does not exist since its left and right hand limits are not equal.

c.

$$\lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$$

d. Since $\lim_{x\to 1} g(x) = 0$ and g is in the denominator, the given limit does not exist.

e.

$$\lim_{x\to 2} x^3 f(x) = \left(\lim_{x\to 2} x^3\right)\left(\lim_{x\to 2} f(x)\right) = 2^3 \cdot 2 = 16$$

f.

$$\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$$

4.

$$\begin{aligned}\lim_{x\to 2}\frac{2x^2+1}{x^2+6x-4} &= \frac{\lim_{x\to 2}(2x^2+1)}{\lim_{x\to 2}(x^2+6x-4)} \\ &= \frac{2\lim_{x\to 2} x^2+\lim_{x\to 2} 1}{\lim_{x\to 2} x^2+6\lim_{x\to 2} x-\lim_{x\to 2} 4} \\ &= \frac{2(2)^2+1}{(2)^2+6(2)-4} = \frac{9}{12} = \frac{3}{4}\end{aligned}$$

The first equality is by limit law 5. The second is by limit laws 1-3. The third is by limit laws 7-9.

8. a. The left-hand side of the equation is not defined for $x=2$, but the right-hand side is.

b. Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \to 2$, just as in Example 3. Remember that in finding $\lim_{x\to a} f(x0$, we never consider $x = a$.

12.

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1}\frac{x^2+x+1}{x+1} = \frac{1^2+1+1}{1+1} = \frac{3}{2}$$

18.

$$\begin{aligned}\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h} &= \lim_{h\to 0}\tfrac{\frac{1}{3+h}-\frac{1}{3}}{h}\\ &= \lim_{h\to 0}\tfrac{3-(3+h)}{h(3+h)3}\\ &= \lim_{h\to 0}\tfrac{-h}{h(3+h)3}\\ &= \lim_{h\to 0}\left(-\tfrac{1}{3(3+h)}\right)\\ &= -\tfrac{1}{\lim_{h\to 0}3(3+h)}\\ &= -\tfrac{1}{3(3+0)} = -\tfrac{1}{9}\end{aligned}$$

32. Since $|x| = x$ for $x > 0$, we have

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x}\right) = \lim_{x\to 0^+} 0 = 0$$

34. a.

$$\lim_{x\to 1^+}\frac{x^2-1}{|x-1|} = \lim_{x\to 1^+}\frac{x^2-1}{x-1} = \lim_{x\to 1^+}(x+1) = 2$$

$$\lim_{x\to 1^-}\frac{x^2-1}{|x-1|} = \lim_{x\to 1^-}\frac{x^2-1}{-(x-1)} = -\lim_{x\to 1^-}(x+1) = -2$$

b. No, $\lim_{x\to 1} F(x)$ does not exist since $\lim_{x\to 1^-} F(x) \neq \lim_{x\to 1^-} F(x)$.
c. See the picture for this question.

36. a. See the picture for this question.
b.

$$\lim_{x\to n^-} f(x) = \lim_{x\to n^-}(x-[x]) = \lim_{x\to n^-}(x-(n-1)) = n-(n-1) = 1$$

$$\lim_{x\to n^+} f(x) = \lim_{x\to n^+}(x-[x]) = \lim_{x\to n^+}(x-n) = n-n = 0$$

c. $\lim_{x\to a} f(x)$ exists is equivalent to a is not an integer.