Mathematics 1a, Section 2.2 Solutions

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2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 8. Since the left-hand and right-hand limits aren't equal, the limit as x approaches 1 of $f(x)$ does not exist.

4. a.

$$\lim_{x \to 0} f(x) = 3$$

b.

$$\lim_{x \to 3^-} f(x) = 4$$

c.

$$\lim_{x \to 3^+} f(x) = 2$$

d. Since the limits in parts **b** and **c** are not equal, the limit does not exist.
e. $f(3) = 3$

8.

$$\lim_{t \to 12^-} f(t) = 150\text{mg}$$
$$\lim_{t \to 12^+} f(t) = 300\text{mg}$$

These limits show that there is a jump in the amount of the drug in the patient's bloodstream at $t = 12$h. The left-hand limit represents the amount of drug just before the injection, and the right-hand limit represents the amount just after.

10. See the picture for this question.

$$\lim_{x\to 0^-} f(x) = 1$$
$$\lim_{x\to 0^+} f(x) = -1$$
$$\lim_{x\to 2^-} f(x) = 0$$
$$\lim_{x\to 0^+} f(x) = 1$$

$f(2) = 1$, and $f(0)$ is undefined.

14. For $g(x) = x\ln(x + x^2)$:

x	$g(x)$
1	-0.693147
0.5	-0.143841
0.1	-0.220727
0.05	-0.147347
0.01	-0.045952
0.005	-0.026467
0.001	-0.006907

It appears that

$$\lim_{x\to 0^+} x\ln(x + x^2) = 0$$

18. For the curve $y = 2^x$ and the points $P(0, 1)$ and $Q(x, 2^x)$:

x	Q	m_{PQ}
0.1	(0.1,1.0717735)	0.71773
0.01	(0.01,1.0069556)	0.69556
0.001	(0.001,1.0006934)	0.69339
0.0001	(0.0001,1.0000693)	0.69317

The slope appears to be about 0.693.

20. $h(x) = \frac{\tan x - x}{x^3}$

a.

x—$h(x)$	
1.0	0.57740773
0.5	0.37041992
0.1	0.33467209
0.05	0.33366700
0.01	0.33334667
0.005	0.33333667

b. It seems that

$$\lim_{x\to 0} h(x) = \frac{1}{3}$$

c. For this chart, the values may value from one calculator to another. Every calculator will eventually give false values.

x	$h(x)$
0.001	0.33333350
0.0005	0.33333344
0.0001	0.33333000
0.00005	0.33333600
0.00001	0.33300000
0.000001	0.00000000

d. As in part **c**, when we take a small enough viewing rectangle we get incorrect output. See the pictures for this question.

22. a. Let $y = \frac{x^3-1}{\sqrt{x}-1}$. See the picture for this question.

x	y
0.99	5.92531
0.999	5.99250
0.9999	5.99925
1.01	6.07531
1.001	6.00750
1.0001	6.00075

From the table and the graph, we guess that the limit of y as x approaches 1 is 6.
b. We need to have $5.5 < \frac{x^3-1}{\sqrt{x}-1} < 6.5$. From the graph we obtain the approximate points of intersection $P(0.9313853, 5.5)$ and $Q(1.0649004, 6.5)$. Now $1 - 0.9313853 \approx 0.0686$ and $1.0649004 - 1 \approx 0.0649$, so by requiring that x be within 0.0649 of 14, we ensure that y is within 0.5 of 6.