Mathematics 1a, Section 2.1 Solutions

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2. To find the heart rate in beats per minute between times t_1 and t_2, we use the formula

$$m = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

where $h(t)$ is the number of heartbeats by time t, as shown in the table. Applying this to the four cases, we get:

t_1	t_2	m
36	42	$\frac{2948-2530}{42-36} = 69.67$
38	42	$\frac{2948-2661}{42-38} = 71.75$
40	42	$\frac{2948-2806}{42-40} = 71$
42	44	$\frac{3080-2948}{44-42} = 66$

We can conclude that the heart rate is decreasing from 71 to 66 heart beats per minute after 42 minutes. After having been stable, the rate is now dropping.

4. a. The table is:

x	Q	m_{PQ}
1.5	(1.5, 0.405465)	0.575364
1.9	(1.9, 0.641854)	0.512933
1.99	(1.99, 0.688135)	0.501254
1.999	(1.999, 0.692647)	0.500125
2.5	(2.5, 0.916291)	0.446287
2.1	(2.1, 0.741937)	0.487902
2.01	(2.01, 0.698135)	0.498754
2.001	(2.001, 0.693647)	0.499875

b. The slope appears to be $\frac{1}{2}$.

c. $y - \ln 2 = \frac{1}{2}(x-2)$.

d. See picture.

5. a. At $t = 2$, $y = 40(2) - 16(2)^2 = 16$. The average velocity between $t = 2$ and $t = 2 + h$ is

$$\frac{40(2+h)^2 - 16(2+h)^2 - 16}{h} = \frac{-24h - 16h^2}{h} = -24 - 16h$$

for $h \neq 0$. We can use this formula at the values of h given:

h	v, ft/s
0.5	-32
0.1	-25.6
0.05	-24.8
0.01	-24.16

b. As $h \to 0$, the average velocity approaches $-24 ft/s$.

6. a. If $h \neq 0$, as in the last problem, we can compute the average velocity between times t and $t + h$.

$$\frac{58(t+h) - 0.83(t+h)^2 - (58t - 0.83t^2)}{h} = \frac{58h - 1.66th - 0.83h^2}{h} = 58 - 1.66t - 0.83h$$

Now we look at the cases where $t = 1$, so the average velocity becomes

$$58 - 1.66 - 0.83h = 56.34 - 0.83h$$

The average velocities for the various intervals are

h	v, m/s
1	55.51
0.5	55.925
0.1	56.257
0.01	56.3317
0.001	56.33917

b. The instantaneous velocity after $1s$ is $56.34 m/s$.

8. a. The average velocity between times $t = 2$ and $t = 2 + h$ is given by

$$v_{av} = \frac{s(2+h) - s(2)}{h}$$

h	v_{av}
3	$\frac{s(5)-s(2)}{5-2} = \frac{178-32}{3} = 48.7\text{ft/s}$
2	$\frac{s(4)-s(2)}{4-2} = \frac{119-32}{2} = 43.5\text{ft/s}$
1	$\frac{s(3)-s(2)}{3-2} = \frac{70-32}{1} = 38\text{ft/s}$

b. See picture. Using the points $(0.8, 0)$ and $(5, 118)$ from the approximate tangent line, the instantaneous velocity at $t = 2$ is about $\frac{118-0}{5-0.8} = 28\text{ft/s}$.

9. a. For the curve $y = \sin(10\pi/x)$ and the point $P(1, 0)$:

x	Q	m_{PQ}
0.5	(0.5,0)	0
0.6	(0.6,0.8660)	-2.1651
0.7	(0.7,0.7818)	-2.6061
0.8	(0.8,1)	-5
0.9	(0.9,-0.3420)	3.4202
1.1	(1.1,-0.2817)	-2.8173
1.2	(1.2,0.8660)	4.3301
1.3	(1.3,-0.8230)	-2.7433
1.4	(1.4,-0.4339)	-1.0847
1.5	(1.5,0.8660)	1.7321
2	(2.0)	0

As x approaches 1, the slopes do not appear to be approaching any particular value.

b. See picture. We see that problems with estimation are caused by the frequent oscillations of the graph. The tangent is so steep at P that we need to take x-values much closer to 1 in order to get accurate estimates of its slope.

c. If we choose $x = 1.001$, then the point Q is $(1.001, -0.0314)$ and $m_{PQ} = -31.3794$. If $x = 0.999$, then Q is $(0.999, 0.0314)$ and $m_{PQ} = -31.4422$. The average of these slopes is -31.4108. So we estimate that the slope of the tangent line at P is about -31.4.