The Substitution Rule

1. Evaluate the following integrals.

   (a) $\displaystyle\int_{-5}^{5} \frac{x^2 - 2x^4}{x^3 + x}\, dx$

   (b) $\displaystyle\int_{0}^{10} |x - 4|\, dx$

   (c) $\displaystyle\int (6x^2 + 2)\sin(x^3 + x + 1)\, dx$

   (d) $\displaystyle\int (1 + x^3)^{3/2} x^2\, dx$

   (e) $\displaystyle\int_{-\pi}^{\pi} x^2 \sin 7x\, dx$

   (f) $\displaystyle\int x^2 e^{x^3}\, dx$

   (g) $\displaystyle\int \frac{\tan^{-1} x}{1 + x^2}\, dx$

   (h) $\displaystyle\int_{0}^{\pi/4} \tan x\, dx$

   (i) $\displaystyle\int \frac{1 + x}{1 + x^2}\, dx$

   (j) $\displaystyle\int \frac{25}{25 + 9x^2}\, dx$

   (k) $\displaystyle\int_{0}^{4} \frac{x}{\sqrt{1 + 2x}}\, dx$

   (l) $\displaystyle\int_{0}^{1} x\sqrt{1 - x^4}\, dx$

2. Suppose that $\displaystyle\int_{0}^{12} g(x)\, dx = \frac{\pi}{12}$. Evaluate $\displaystyle\int_{0}^{3} g(4x)\, dx$.

3. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 liters per second. This explains, in part, why the function

   $$f(t) = \frac{1}{2}\sin\left(\frac{2\pi t}{5}\right)$$

   has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t assuming there was no air in the lungs at time $t = 0$.